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Using Min-max Method to Solve a Full Fuzzy Linear Programming

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Abstract: In this paper, we propose a new procedure to solve a full fuzzy linear programming such that all parameters and variables in the model are triangular fuzzy numbers. First, we approximate all fuzzy numbers by the nearest symmetric triangular fuzzy numbers. Then, using arithmetic of fuzzy numbers, we have a multiobjective linear programming (MOLP) where the center and margin of fuzzy numbers are considered as objective functions in our MOLP. Also, all parameters and variables in this MOLP are crisp. After that, MOLP is solved by min-max method. We prefer min-max method to lexicography method proposed by Hosseinzadeh *et al.* (2009), due to the fact that min-max method considers the center and margin of fuzzy number simultaneously, while lexicography method prefers the center of a fuzzy number to its margin. Finally, Numerical examples show that the solution of full fuzzy linear programming using min-max method has less margin than lexicography method.

Key words: fuzzy numbers; linear programming; multiobjective linear programming (MOLP); min-max method.

INTRODUCTION

Subject of decision making in a fuzzy environment was first introduced by Zadeh *et al*. (1970). Many authors have proposed various methods to solve fuzzy linear programming (Wang,, 1997; Allahviranloo *et al.,* 2007; Maleki, 2002; Mahdavi-Amiri, S. H. Nasseri, 2006; Mishmast Nehi *et al.,* 2004; Maleki *et al.,* 2000; Van Hop, 2007; Garcia-Aguado and Verdegay, 1993; Li *et al.,* 2004; Negoita, 1970; Buckley, 1983; Mahadavi-Amiri, S. H. Nasseri, 2007). However, many of them had problems when all of the parameters and variables of fuzzy linear programming (FLP) were assumed to be fuzzy, e.g. at the same time, the right hand side and the objective function coefficients and the variables were not assumed to be fuzzy. Garcia-Aguado *et al*. (1993) showed that in the case of fuzzy linear programming problems, whether or not a fuzzy optimal solution had been found using linear membership functions modeling the constraints, possible further changes of those membership functions did not affect the former optimal solution. Buckley, (1983) presented that how fuzzy programming, using either the min or product operator, may be used to generate the whole Pareto optimal set for nonlinear concave, or convex, multiobjective programming problems. Also, a new solution procedure for a fuzzy program, when the min operator was employed, was presented. Mahdavi-Amiri *et al*. (2006) explored some duality properties in fuzzy number linear programming problems. Using a linear ranking function, they introduced the dual of fuzzy number linear programming primal problems. Mahdavi-Amiri *et al*. (2007) applied a linear ranking function to order trapezoidal fuzzy numbers. Then, they established the dual problem of the linear programming problem with trapezoidal fuzzy variables and hence deduced some dual results. In particular, they proved that the auxiliary problem was indeed the duality of the FVLP problem. Having established the dual problem, the results will then follow as natural extensions of duality results for linear programming problems with crisp data. Nguyen Van Hop, (2007) presented a model to measure the superiority and inferiority of fuzzy numbers/fuzzy stochastic variables. Then, the new measures were used to convert the fuzzy (stochastic) linear program into the corresponding deterministic linear program. Recently, Hosseinzadeh *et al*. (2009) proposed a new method to solve a full fuzzy linear programming (FFLP), in such way that: first, the triangular fuzzy number was approximated to its nearest symmetric triangular fuzzy number. Then, their FLP was transformed to a MOLP by arithmetic of fuzzy numbers in which all of the parameters and variables of the MOLP are crisp numbers, in terms of the core and fuzziness. They solved the MOLP by lexicography method, because they preferred the core of solution to margins, while we know that preferences in the core of solution and margins are the same. Even in some conditions, the margins of solution may be

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preferred to the core. This paper is proposed to show a new way to solve the FFLP by min-max method and using nearest symmetric triangular defuzzification proposed by Ezzati *et al*. (2010) to approximate all triangular fuzzy numbers. Our method has considered the core and margin of symmetric triangular fuzzy number at the same time in the objective functions. By two examples, we are showing that our method has less margin than the proposed method in Hosseinzadeh Lotfi *et al*., (2009). Besides, our procedure is computationally easier than the proposed method in LP to solve the FFLP, where they used two LP's to solve it. On the other hand, it will be shown that our optimal value is more desirable than the obtained optimal value by them.

This paper is organized as follows: In Section 2, we note some definitions and notations of fuzzy numbers, and then we introduce the nearest parametric symmetric triangular defuzzification proposed by Ezzati *et al*. (2010). Then we use the min-max method to solve the MOLP using defuzzification proposed by Ezzati *et al*. (2010), the FFLP is transformed to the MOLP with two objective functions in Section 3. In Section 4 two examples are given and the optimal value of our method will be compared with the optimal value obtained in Hosseinzadeh Lotfi *et al*., (2009) and finally the conclusion is drawn in Section 5.

2 Basic Definitions and Notations:

The basic definitions of a fuzzy number are given in Hosseinzadeh Lotfi *et al*., (2009) as follows.

We represent an arbitrary fuzzy number by an ordered pair of functions $\tilde{u} = (\underline{u}(r), \overline{u}(r))$, $0 \le r \le 1$, where it satisfies the following requirement:

1. $\underline{u}(r)$ is a bounded left continuous nondecreasing function over [0,1].
2. $\overline{u}(r)$ is a bounded left continuous nonincreasing function over [0,1].
3. $\underline{u}(r)$ and $\overline{u}(r)$ are right continuous at 0.
4. $\underline{u}(r) \leqslant \overline{u}(r), 0 \leqslant r \leqslant 1.$

A crisp number α is simply represented by $\underline{u}(r) = \overline{u}(r) = \alpha, 0 \le r \le 1.$

If $\underline{u}(1) = \overline{u}(1)$, then $\tilde{u}$ is triangular fuzzy number. In this paper, we denote the set of all triangular fuzzy numbers by $\tilde{T}$.

Definition 2.1:

$C_{\tilde{u}} = Core(\tilde{u}) = \overline{u}(1)$; and $W_{\tilde{u}}^{L} = C_{\tilde{u}} - \underline{u}(0) \geqslant 0$ and $W_{\tilde{u}}^{R} = \overline{u}(0) - C_{\tilde{u}} \geqslant 0$ are the left and right margins of u , respectively.

Definition 2.2:

The fuzzy number

$$\tilde{t} =: (C_{\tilde{t}} - W_{\tilde{t}}^{L} + W_{\tilde{t}}^{L} r, C_{\tilde{t}} + W_{\tilde{t}}^{R} - W_{\tilde{t}}^{R} r) =: (C_{\tilde{t}}, W_{\tilde{t}}^{L}, W_{\tilde{t}}^{R}),\ 0 \leqslant r \leqslant 1,$$

is an asymmetric triangular fuzzy number. As a matter of fact $C_{\tilde{t}} - W_{\tilde{t}}^{L} + W_{\tilde{t}}^{L} r = \underline{t}(r)$ and $C_{\tilde{t}} + W_{\tilde{t}}^{R} - W_{\tilde{t}}^{R} r = \overline{t}(r)$ where $C_{\tilde{t}}, W_{\tilde{t}}^{L}$ and $W_{\tilde{t}}^{R} \in \Re$. Let $\tilde{A}$ be the set of all asymmetric triangular fuzzy numbers.

A conventional fuzzy number is the symmetric triangular fuzzy numbe $S[x_o, \sigma]$, where $W_S^L = W_S^R = \sigma$ centered at x_o with basis 2σ . Its parametric form is

$$S[x_o, \sigma] =: (x_o - \sigma + r\sigma, x_o + \sigma - r\sigma) := (x_o; \sigma),$$

$0 \leqslant r \leqslant 1$ which x_o , $\sigma \in \Re$ and x_o is the center and $\sigma \geqslant 0$ is the margin of $S[x_o,\sigma]$. $S[x_o,\sigma]$ is called symmetric triangular fuzzy number.

Suppose that $\tilde{S}$ be the set of all symmetric triangular fuzzy numbers.

Definition 2.3.:

Let $\tilde{t} = (C_1, W_1^L, W_1^R), \tilde{u} = (C_2, W_2^L, W_2^R)$. Using extension principal we can define:

1. $\tilde{t} = \tilde{u}$ if and only if $W_1^L = W_2^L$ and $W_1^R = W_2^R$
2. $\tilde{t} + \tilde{u} = (C_1 + C_2, W_1^L + W_2^L, W_1^R + W_2^R)$.
3. $k\tilde{t} = \begin{cases} (kC_1, kW_1^L, kW_1^R) & when\ k \geqslant 0, \\ (kC_1, -kW_1^R, -kW_1^L) & when\ k \leqslant 0. \end{cases}$

Definition 2.4:

For two fuzzy numbers in parametric forms $\tilde{t} = (\underline{t}(r), \overline{t}(r)), \tilde{u} = (\underline{u}(r), \overline{u}(r)),$ we have

$$\tilde{t}\tilde{u} = \tilde{h} = (\underline{h}(r), \overline{h}(r)),$$

where

$$\underline{h}(r) = \min\{\underline{t}(r)\underline{u}(r), \overline{t}(r)\overline{u}(r), \overline{t}(r)\underline{u}(r), \underline{t}(r)\overline{u}(r)\}$$

and

$$\overline{h}(r) = \max\{\underline{t}(r)\underline{u}(r), \overline{t}(r)\overline{u}(r), \overline{t}(r)\underline{u}(r), \underline{t}(r)\overline{u}(r)\}.$$

For example, for two positive asymmetric triangular fuzzy numbers

$\tilde{t} = (C_{\tilde{t}} + W_{\tilde{t}}^L(r-1), C_{\tilde{t}} + W_{\tilde{t}}^R(1-r))$ and $\tilde{u} = (C_{\tilde{u}} + W_{\tilde{u}}^L(r-1), C_{\tilde{u}} + W_{\tilde{u}}^R(1-r))$ where $C_{\tilde{t}} - W_{\tilde{t}}^L \geqslant 0$ and $C_{\tilde{u}} - W_{\tilde{u}}^L \geqslant 0,$ one can obtain $\tilde{t}\tilde{u}$ as follows:

$$\tilde{t}\tilde{u} = (f(r), g(r))$$

where

$$f(r) = C_{\tilde{t}}C_{\tilde{u}} + C_{\tilde{t}}W_{\tilde{u}}^L(r-1) + W_{\tilde{t}}^L(r-1)C_{\tilde{u}} + W_{\tilde{t}}^L W_{\tilde{u}}^L(r-1)^2,$$

$$g(r) = C_{\tilde{t}}C_{\tilde{u}} + C_{\tilde{t}}W_{\tilde{u}}^R(1-r) + W_{\tilde{t}}^R(1-r)C_{\tilde{u}} + W_{\tilde{t}}^R W_{\tilde{u}}^R(r-1)^2.$$

Suppose $\tilde{A} \in E^{n^2}$,E is the Euclidian space of fuzzy numbers. Also, suppose that $\tilde{X} = (\tilde{t}_1, \tilde{t}_2, ..., \tilde{t}_n)^T$ and $\tilde{Y} = (\tilde{u}_1, \tilde{u}_2, ..., \tilde{u}_n)^T$ be two asymmetric triangular fuzzy vectors. Now we have

1. $Core(\tilde{X} + \tilde{Y}) = Core(\tilde{X}) + Core(\tilde{Y}),$

2. $Core(\tilde{A}\tilde{X}) = Core(\tilde{A})Core(\tilde{X}),$

3. $\tilde{A}(\tilde{X}+\tilde{Y}) = \tilde{A}\tilde{X}+\tilde{A}\tilde{Y}.$

Definition 2.5. (Ezzati et al., 2010):

Let $\tilde{A}$ be a fuzzy number and $(\underline{A}(r), \overline{A}(r))$ be its parametric form. The following values constitute the weighted averaged representative and weighted width, respectively, of the fuzzy number $\tilde{A}$:

$$I(\tilde{A}) = \frac{1}{2}\int_0^1 (\underline{A}(r) + \overline{A}(r))dr, (\tilde{A}) = \int_0^1 (\overline{A}(r) - \underline{A}(r)) f(r)dr.$$

The function *f*(*x*) is also called weighting function.

Definition 2.6. (Ezzati et al., 2010):

For arbitrary fuzzy numbers $\tilde{A}$ and $\tilde{B}$ the quantity

$$d_p = \sqrt{[I(\tilde{A}) - I(\tilde{B})]^2 + [D(\tilde{A}) - D(\tilde{B})]^2}$$

is called the parametric distance between the fuzzy numbers $\tilde{A}$ and $\tilde{B}$.

2.1. Nearest Parametric Symmetric Triangular Defuzzification:

Let $\tilde{A}$ be a general fuzzy number and $(\underline{A}(r), \overline{A}(r))$ be its parametric form. To obtain a symmetric triangular fuzzy number which is the nearest to $\tilde{A}$, we should minimize:

$$d_p(\tilde{A}, S[x_{0p}, \sigma_p]) = ([I(\tilde{A}) - I(S[x_{0p}, \sigma_p])]^2 + [D(\tilde{A}) - D(S[x_{0p}, \sigma_p])]^2)^{\frac{1}{2}}$$

with respect to x_{0p} and σ_p . If $S[x_{0p}, \sigma_p]$ minimizes $d_p(\tilde{A}, S[x_{0p}, \sigma_p]),$ then it will be provide a defuzzification of $\tilde{A}$ with a defuzzifier x_{0p} and σ_P . In order to minimize $d_p(\tilde{A}, S[x_{0p}, \sigma_p]),$ let $\overline{D}_P(\tilde{A}, S[x_{0p}, \sigma_p]) = d_p^2(\tilde{A}, S[x_{0p}, \sigma_p])$ and then minimize it. We consider:

$$\frac{\partial \overline{D}_P(\tilde{A}, S[x_{0p}, \sigma_p])}{\partial \sigma_p} = 0,$$

$$\frac{\partial \overline{D}_P(\tilde{A}, S[x_{0p}, \sigma_p])}{\partial x_{0p}} = 0.$$

The solution is:

$$\sigma_p = \frac{\int_0^1 (\overline{A}(r) - \underline{A}(r)) f(r)(1-r)dr}{2\int_0^1 f(r)(1-r)^2 dr}, \qquad x_{0p} = \frac{1}{2}\int_0^1 (\underline{A}(r) + \overline{A}(r))dr.$$

3 Full Fuzzy Linear Programming Problem:

Suppose the following FFLP:

$$\begin{cases} \max(C_{\tilde{C}}, W^L_{\tilde{C}}, W^R_{\tilde{C}})(C_{\tilde{X}}, W^L_{\tilde{X}}, W^R_{\tilde{X}}) \\ \\ s.t.\ (C_{\tilde{A}}, W^L_{\tilde{A}}, W^R_{\tilde{A}})(C_{\tilde{X}}, W^L_{\tilde{X}}, W^R_{\tilde{X}}) \simeq (C_{\tilde{b}}, W^L_{\tilde{b}}, W^R_{\tilde{b}}), \\ \quad C_{\tilde{X}} - W^L_{\tilde{X}} \geqslant 0, \\ \quad (C_{\tilde{X}}, W^L_{\tilde{X}}, W^R_{\tilde{X}}) \in \tilde{T}^n, \end{cases}$$

where

$$\tilde{X} = (C_{\tilde{X}}, W^L_{\tilde{X}}, W^R_{\tilde{X}}),\ \tilde{b} = (C_{\tilde{b}}, W^L_{\tilde{b}}, W^R_{\tilde{b}}),\ \tilde{A} = (C_{\tilde{A}}, W^L_{\tilde{A}}, W^R_{\tilde{A}}), \tilde{C} = (C_{\tilde{C}}, W^L_{\tilde{C}}, W^R_{\tilde{C}})$$

$$C_{\tilde{X}} = Core(\tilde{X}), C_{\tilde{b}} = Core(\tilde{b}), C_{\tilde{A}} = Core(\tilde{A}), C_{\tilde{C}} = Core(\tilde{C}),$$

and $W_{\tilde{X}}, W_{\tilde{b}}, W_{\tilde{A}}$ and $W_{\tilde{C}}$ are the margins of $\tilde{X}, \tilde{b}, \tilde{A}$ and $\tilde{C}$, respectively. $\tilde{A} \in E^{m\times n}, \tilde{b} \in E^m, \tilde{C}$ and $\tilde{X} \in E^n$ and also $\tilde{A}, \tilde{C}, \tilde{X}$ and $\tilde{b}$ are arbitrary fuzzy matrix and fuzzy vectors. Here, we have $C_{\tilde{A}} - W^L_{\tilde{A}} \geqslant 0,\ C_{\tilde{C}} - W^L_{\tilde{C}} \geqslant 0,$ and $C_{\tilde{b}} - W^L_{\tilde{b}} \geqslant 0$. Applying the fuzzy production of two positive fuzzy asymmetric parameter numbers, and using definition (2.4), then $(x_{0_{\tilde{C}\tilde{X}}}, \sigma_{\tilde{C}\tilde{X}}), (x_{0_{\tilde{A}\tilde{X}}}, \sigma_{\tilde{A}\tilde{X}})$ and $(x_{0_{\tilde{b}}}, \sigma_{\tilde{b}})$, will be the nearest parametric symmetric triangular fuzzy numbers to $\tilde{C}\tilde{X}, \tilde{A}\tilde{X}$ and $\tilde{b}$, respectively, that are derived from the following relations:

(Note that in the following computations, we assume that $f(r) = r$

$$\sigma_{\tilde{C}\tilde{X}} = \frac{\int_0^1 [(C_{\tilde{C}}W^R_{\tilde{X}} + C_{\tilde{X}}W^R_{\tilde{C}} + C_{\tilde{C}}W^L_{\tilde{X}} + C_{\tilde{X}}W^L_{\tilde{C}})(1-r) + (W^R_{\tilde{C}}W^R_{\tilde{X}} - W^L_{\tilde{C}}W^L_{\tilde{X}})(1-r)^2]r(1-r)dr}{2\int_0^1 r(1-r)^2 dr}$$

$$= 1/2(C_{\tilde{C}}W^R_{\tilde{X}}) + 1/2(C_{\tilde{X}}W^R_{\tilde{C}}) + 3/10(W^R_{\tilde{C}}W^R_{\tilde{X}}) + 1/2(C_{\tilde{C}}W^L_{\tilde{X}}) + 1/2(C_{\tilde{X}}W^L_{\tilde{C}})$$

$$-3/10(W^L_{\tilde{C}}W^L_{\tilde{X}}),$$

$$x_{0_{\tilde{C}\tilde{X}}} = 1/2\int_0^1 [2C_{\tilde{C}}C_{\tilde{X}} + (C_{\tilde{C}}W^L_{\tilde{X}} + C_{\tilde{X}}W^L_{\tilde{C}})(r-1) + W^L_{\tilde{C}}W^L_{\tilde{X}}(r-1)^2 +$$

$$(C_{\tilde{C}}W^R_{\tilde{X}} + C_{\tilde{X}}W^R_{\tilde{C}})(1-r) + W^R_{\tilde{C}}W^R_{\tilde{X}}(r-1)^2]dr$$

$$= C_{\tilde{C}}C_{\tilde{X}} - 1/4C_{\tilde{C}}W^L_{\tilde{X}} - 1/4C_{\tilde{X}}W^L_{\tilde{C}} + 1/6W^L_{\tilde{C}}W^L_{\tilde{X}}$$

$$+1/4C_{\tilde{C}}W^R_{\tilde{X}} + 1/4C_{\tilde{X}}W^R_{\tilde{C}} + 1/6W^R_{\tilde{C}}W^R_{\tilde{X}},$$

and for constrains we have:

$$\sigma_{\tilde{A}\tilde{X}} = \frac{\int_0^1 [(C_{\tilde{A}}W_{\tilde{X}}^R + C_{\tilde{X}}W_{\tilde{A}}^R + C_{\tilde{A}}W_{\tilde{X}}^L + C_{\tilde{X}}W_{\tilde{A}}^L)(1-r) + (W_{\tilde{A}}^R W_{\tilde{X}}^R - W_{\tilde{A}}^L W_{\tilde{X}}^L)(1-r)^2]r(1-r)dr}{2\int_0^1 r(1-r)^2 dr}$$

$$= 1/2(C_{\tilde{A}}W_{\tilde{X}}^R) + 1/2(C_{\tilde{X}}W_{\tilde{A}}^R) + 3/10(W_{\tilde{A}}^R W_{\tilde{X}}^R) + 1/2(C_{\tilde{A}}W_{\tilde{X}}^L) +$$

$$\cdot 1/2(C_{\tilde{X}}W_{\tilde{A}}^L) - 3/10(W_{\tilde{A}}^L W_{\tilde{X}}^L),$$

$$x_{0_{\tilde{A}\tilde{X}}} = 1/2\int_0^1 [2C_{\tilde{A}}C_{\tilde{X}} + (C_{\tilde{A}}W_{\tilde{X}}^L + C_{\tilde{X}}W_{\tilde{A}}^L)(r-1) + W_{\tilde{A}}^L W_{\tilde{X}}^L (r-1)^2 + (C_{\tilde{A}}W_{\tilde{X}}^R +$$

$$C_{\tilde{X}}W_{\tilde{A}}^R)(1-r) + W_{\tilde{A}}^R W_{\tilde{X}}^R (r-1)^2]dr$$

$$= C_{\tilde{A}}C_{\tilde{X}} - 1/4C_{\tilde{A}}W_{\tilde{X}}^L - 1/4C_{\tilde{X}}W_{\tilde{A}}^L + 1/6W_{\tilde{A}}^L W_{\tilde{X}}^L + 1/4C_{\tilde{A}}W_{\tilde{X}}^R + 1/4C_{\tilde{X}}W_{\tilde{A}}^R + 1/6W_{\tilde{A}}^R W_{\tilde{X}}^R$$

For the right hand side of constraint we have:

$$\sigma_{0\tilde{b}} = \frac{\int_0^1 (W_{\tilde{b}}^R + W_{\tilde{b}}^L)(1-r)^2 r\,dr}{2\int_0^1 r(1-r)^2 dr} = 1/2W_{\tilde{b}}^R + 1/2W_{\tilde{b}}^L,$$

$$x_{0\tilde{b}} = 1/2\int_0^1 2c_{\tilde{b}} + (W_{\tilde{b}}^R - W_{\tilde{b}}^L)(1-r)dr = C_{\tilde{b}} + 1/4W_{\tilde{b}}^R - 1/4W_{\tilde{b}}^L.$$

Now consider the FFLP as a MOLP. Suppose that $F_0(\tilde{X}) := (-C_{\tilde{C}\tilde{X}})$ and $F_1(\tilde{X}) := W_{\tilde{C}\tilde{X}}$ and also

$$S = \{\tilde{X} \mid \tilde{A}\tilde{X} \simeq \tilde{b}, C_{\tilde{X}} - W_{\tilde{X}}^L \geqslant 0, \tilde{X} \in \tilde{T}^n\}.$$

Then, we solve the following MOLP:

$$\begin{cases} \min & \{F_0(\tilde{X}), F_1(\tilde{X})\} \\ s.t. & \tilde{X} \in S. \end{cases}$$

Hosseinzadeh *et al*. (2009) proposed a new method to solve a full fuzzy linear programming (FFLP). They solved the MOLP by lexicography method, because they preferred the core of the solution to margins, while we know that the preferences of the core of a solution and the margins are the same. Even in some conditions, the margins of the solution may be preferred to the core. So, we apply the min-max rule to consider the center and the margin of fuzzy numbers simultaneously.The given problem(MOLP) can be shown as follows after applying the min-max method:

$$\begin{cases} \min & Z \\ s.t. & F_0(\tilde{X}) \leqslant Z, \\ & F_1(\tilde{X}) \leqslant Z, \\ & \tilde{X} \in S. \end{cases}$$

After substitute the value of $F_0(\tilde{X})$ and $F_1(\tilde{X})$ according to the above calculates we will have:

$$\begin{cases} \min Z \\ s.t. \\ -C_{\tilde{C}}C_{\tilde{X}} + 1/4C_{\tilde{C}}W_{\tilde{X}}^{L} + 1/4C_{\tilde{X}}W_{\tilde{C}}^{L} - 1/6W_{\tilde{C}}^{L}W_{\tilde{X}}^{L} - 1/4C_{\tilde{C}}W_{\tilde{X}}^{R} - \\ 1/4C_{\tilde{X}}W_{\tilde{C}}^{R} - 1/6W_{\tilde{C}}^{R}W_{\tilde{X}}^{R} \leqslant Z, \\ 1/2(C_{\tilde{C}}W_{\tilde{X}}^{R}) + 1/2(C_{\tilde{X}}W_{\tilde{C}}^{R}) + 3/10(W_{\tilde{C}}^{R}W_{\tilde{X}}^{R}) + 1/2(C_{\tilde{C}}W_{\tilde{X}}^{L}) + \\ 1/2(C_{\tilde{X}}W_{\tilde{C}}^{L}) - 3/10(W_{\tilde{C}}^{L}W_{\tilde{X}}^{L}) \leqslant Z, \\ C_{\tilde{A}}C_{\tilde{X}} - 1/4C_{\tilde{A}}W_{\tilde{X}}^{L} - 1/4C_{\tilde{X}}W_{\tilde{A}}^{L} + 1/6W_{\tilde{A}}^{L}W_{\tilde{X}}^{L} + 1/4C_{\tilde{A}}W_{\tilde{X}}^{R} + \\ 1/4C_{\tilde{X}}W_{\tilde{A}}^{R} + 1/6W_{\tilde{A}}^{R}W_{\tilde{X}}^{R} = C_{\tilde{b}} + 1/4W_{\tilde{b}}^{R} - 1/4W_{\tilde{b}}^{L} + 1/2(C_{\tilde{A}}W_{\tilde{X}}^{R}) \\ +1/2(C_{\tilde{X}}W_{\tilde{A}}^{R}) + 3/10(W_{\tilde{A}}^{R}W_{\tilde{X}}^{R}) + 1/2(C_{\tilde{A}}W_{\tilde{X}}^{L}) + 1/2(C_{\tilde{X}}W_{\tilde{A}}^{L}) - 3/10(W_{\tilde{A}}^{L}W_{\tilde{X}}^{L}) = \\ 1/2W_{\tilde{b}}^{R} + 1/2W_{\tilde{b}}^{L}, \\ C_{\tilde{X}} - W_{\tilde{X}}^{L} \geqslant 0, \\ W_{\tilde{X}}^{L} \geqslant 0, \\ W_{\tilde{X}}^{R} \geqslant 0. \end{cases}$$

Theorem 3.1.:

$\tilde{X}^{*} = (C_{\tilde{X}^{*}}, W_{\tilde{X}^{*}}^{L}, W_{\tilde{X}^{*}}^{R})$ is an optimal solution of the FFLP if $(C_{\tilde{X}^{*}}, W_{\tilde{X}^{*}}^{L}, W_{\tilde{X}^{*}}^{R})$ is an optimal solution of the min-max problem.

Proof:

The proof is the same as Theorem 3.1 in Hosseinzadeh *et al*. (2009).

4 Examples:

Example. 4.1. Hosseinzadeh et al. (2009):

Consider the FFLP where:

$$\tilde{C}=\begin{bmatrix}(15,5,2)\\(16,6,4)\\(14,4,3)\\(12,2,2)\end{bmatrix},\ \tilde{A}=\begin{bmatrix}(10,2,3) & (11,1,2) & (12,3,1) & (15,4,2)\\(14,2,2) & (18,4,1) & (17,3,3) & (14,1,4)\end{bmatrix},$$

$$\tilde{b}=\begin{bmatrix}(411.75,140,162)\\(539.5,154,220)\end{bmatrix}.$$

Suppose $\tilde{X}_1=(x_1,x'_1,x''_1), \tilde{X}_2=(x_2,x'_2,x''_2), \tilde{X}_3=(x_3,x'_3,x''_3)$ and $\tilde{X}_4=(x_4,x'_4,x''_4)$. We need to solve the

$$\begin{aligned}
&\min = Z\\
&s.t.\\
&10.25x_1+11.25x_2+11.5x_3+14.5x_4-2.17x'_1-2.58x'_2-2.5x'_3-3.08x'_4+3x''_1+3.08x''_2+\\
&3.17x''_3+4.08x''_4=417.25,\\
&14x_1+17.25x_2+17x_3+14.75x_4-3.16x'_1-3.83x'_2-3.75x'_3-3.3x'_4+3.83x''_1+4.66x''_2+\\
&4.75x''_3+4.16x''_4=556,\\
&2.5x_1+1.5x_2+2x_3+3x_4+4.25x'_1+5.125x'_2+4.875x'_3+6x'_4+6.125x''_1+6.25x''_2+\\
&6.375x''_3+8.25x''_4=151,\\
&2x_1+2.5x_2+3x_3+2.5x_4+6.25x'_1+7.5x'_2+7.375x'_3+6.625x'_4+7.75x''_1+9.375x''_2+\\
&9.625x''_3+8.5x''_4=187,\\
&-14.25x_1-15.5x_2-12.25x_3-12x_4+2.9x'_1+3x'_2+2.8x'_3+2.7x'_4-4.1x''_1-4.7x''_2-\\
&4x''_3-3.3x''_4\leqslant Z,\\
&3.5x_1+5x_2+3.5x_3+2x_4+6x'_1+6.2x'_2+5.8x'_3+5.4x'_4+8.1x''_1+9.2x''_2+7.9x''_3+6.6x''_4\leqslant Z,\\
&x_i-x'_i\geqslant 0,\quad i=1,2,3,4,\\
&x_i\geqslant 0,\quad i=1,2,3,4,\\
&x(i)\geqslant 0,\ \ i=1,2,3,4,\\
&Z\geqslant 0.
\end{aligned}$$

Solving this problem, leads to $C_{\tilde{C}\tilde{X}}(\tilde{X})=412, W_{\tilde{C}\tilde{X}}(\tilde{X})=176.23$ and

$$\tilde{X}^*=\begin{bmatrix}(0,0,0)\\(0,0,0)\\(25.46,5.58,0)\\(8.03,0,5.85)\end{bmatrix}.$$

The solution of lexicography rule is:

$$C_{\tilde{C}\tilde{X}}(\tilde{X})=521,\quad W_{\tilde{C}\tilde{X}}(\tilde{X})=236.$$

Obviously, our result has less margins value than that of lexicography rule presented in Hosseinzadeh *et al*. (2009), and also our procedure is computationally easier.

Example. 4.2:

In fact, some of the data in this example is borrowed from (Dehghan, Behnam Hashemi, 2006). A manufacturing company that makes three types of computers: *A*, *B* and *C*.

Computer *A* takes about $\tilde{19} = (19,1,1)$ hours for assembling (the ingredients), $\tilde{2} = (2,0.1,0.1)$ hours for testing (the hardware), $\tilde{2} = (2,0.1,0.2)$ hours for installing (the software). Computer *B* takes about $\tilde{12} = (12,1.5,1.5)$ hours for assembling (the ingredients), $\tilde{4} = (4,0.1,0.4)$ hours for testing (the hardware), $\tilde{2} = (2,0.1,0.3)$ hours for installing (the software). Computer *C* takes about $\tilde{6} = (6,0.5,0.2)$ hours for assembling (the ingredients), $\tilde{1.5} = (1.5,0.2,0.2)$ hours for testing (the hardware), $\tilde{4.5} = (4.5,0.1,0.1)$ hours for installing (the software). The company has a factory which works about $\tilde{1897} = (1897,427.7,536.2)$ labor-hours each month for assembling, $\tilde{434.5} = (434.5,76.2,109.3)$ labor-hours for testing, and $\tilde{535.5} = (535.5,88.3,131.9)$ hours for installing. The cost of ingredients of these computers is $\tilde{20} = (20,5,2)$, $\tilde{25} = (25,6,4)$ and $\tilde{30} = (30,4,3)$ for *A,B* and *C*, respectively. How many computers of each kind with the minimum total cost can the factory make in a month?

Let $\tilde{x} = (x_1, y_1, z_1)$, $\tilde{y} = (x_2, y_2, z_2)$ and $\tilde{z} = (x_3, y_3, z_3)$ show the number of computers of type *A,B* and *C*, respectively. We need to solve the following problem:

$$\begin{cases} \min Z \\ \\ s.t. \\ -19.25x_1 - 24.5x_2 - 29.75x_3 - 5.33z_1 - 6.91z_2 - 8z_3 + 4.17y_1 + 5.25y_2 + 6.83y_3 \le Z, \\ 3.5x_1 + 5x_2 + 3.5x_3 + 8.5y_1 + 10.7y_2 + 13.8y_3 + 10.6z_1 + 13.7z_2 + 15.9z_3 \le Z, \\ 19x_1 + 12x_2 + 5.925x_3 - 4.58y_1 - 2.75y_2 - 1.42y_3 + 4.92z_1 + 3.75z_2 + 1.53z_3 = 1924.125, \\ 2x_1 + 4.075x_2 + 1.5x_3 - 0.483y_1 - 0.983y_2 - 0.342y_3 + 0.517z_1 + 1.067z_2 + 0.379z_3 = 442.775, \\ 2.025x_1 + 2.05x_2 + 4.5x_3 - 0.48y_1 - 0.48y_2 - 1.108y_3 + 0.533z_1 + 0.55z_2 + 1.1417z_3 = 546.4, \\ x_1 + 1.5x_2 + 0.35x_3 + 9.2y_1 + 5.55y_2 + 2.85y_3 + 9.8z_1 + 6.45z_2 + 3.6z_3 = 481.95, \\ 0.1x_1 + 0.25x_2 + 0.2x_3 + 0.97y_1 + 1.97y_2 + 0.69y_3 + 1.03z_1 + 2.12z_2 + 0.81z_3 = 92.75 \\ 0.15x_1 + 0.2x_2 + 0.1x_3 + 0.97y_1 + 0.97y_2 + 2.22y_3 + 1.06z_1 + 1.09z_2 + 2.28z_3 = 110.1, \\ x_1 - y_1 \ge 0, \\ x_2 - y_2 \ge 0, \\ x_3 - y_3 \ge 0, \\ Z \ge 0, x_i \ge 0, y_i \ge 0, \ i = 1,2,3. \end{cases}$$

The solution for this problem is:

$$\begin{bmatrix} (x_1^*, y_1^*, z_1^*) \\ (x_2^*, y_2^*, z_2^*) \\ (x_3^*, y_3^*, z_3^*) \end{bmatrix} = \begin{bmatrix} (43.54544, 20.76889, 0) \\ (64.01867, 9.87663, 0) \\ (81.7942, 23.8102, 0) \end{bmatrix}.$$

Also, we have:

$$Z^* = W_{\tilde{C}\tilde{X}}(\tilde{X}) = 1369.580, \quad C_{\tilde{C}\tilde{X}}(\tilde{X}) = 4539.00234,$$

Therefore, the optimal objective value of this problem is a fuzzy number as

(4539.00234,1369.580,1369.580).

Conclusion:

In this paper, we have proposed a new procedure to solve full fuzzy linear programming such that all parameters and variables in the model are triangular fuzzy numbers. The FFLP has been changed to a MOLP using new defuzzification, which was proposed by Ezzati *et al*. (2010), and also arithmetic of fuzzy numbers. After that, the min-max method has been used to transform the MOLP to LP. One of the reasons of using the min-max method is that, the core and the margin of fuzzy numbers should be considered at the same time in the objective functions. Besides, our procedure is computationally easier than the one Hosseinzadeh Lotfi *et al*., (2009), since we have applied only one LP to solve the FFLP, while they had used two LP's to solve it. Finally, we have shown with two examples that our optimal value has less marginal score than the solution of lexicography method presented in Hosseinzadeh Lotfi *et al*., (2009). Therefore, it can be said that our method is more efficient than that of Hosseinzadeh Lotfi *et al*., (2009) to solve the FFLP.

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