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An Application of Mathematical Model to Time-cost Trade off Problem (Case Study)

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Abstract: Time and cost are two main criteria that should be considered carefully in project scheduling. Project managers are always trying to determine an economic time in which total cost of project would be minimum. In this paper, a company is chosen for study that it has received a new order for constructing a subassembly part for a farming machine. In order to contract with this customer, the managers of the company have decided to implement project management techniques for estimating the project's time and cost. In the first step, the list of project activities and the relationships between them are determined. Afterwards, the normal and crash duration are estimated for each activity as well as the normal and crash cost (direct costs). Finally, indirect costs of project are estimated per each day of project. In the second step, an appropriate mathematical model is chosen from the literature review for this problem and it is solved with LINGO 12.0. As a result, the early start and budgeted cost for each activity, the total indirect costs of project, and completion time of project are calculated so that the total costs of project remain in the minimum level.

Key words: Time-cost trade off, mathematical programming, project management

INTRODUCTION

An important aspect of project management is scheduling time accurately. This is a critical component of Project planning as this will decide the deadline for the completion of a project - whether small, medium or mega (Suri *et al*., 2009). Since the late 1950s, Critical-Path-Method (CPM) techniques have become widely recognized as valuable tools for the planning and scheduling of projects. But in many cases, project should implement before the date that was calculated by CPM method. Achieving this goal, can be used more productive equipment or hiring more workers (Leu *et al*., 2001). Reducing the original project duration which is called crashing PERT/CPM networks' in many studies which is aimed at meeting a desired deadline with the lowest amount of cost is one of the most important and useful concepts for project managers. Since there is a need to allocate extra resources in PERT/CPM crashing networks, and the project managers are intended to spend the lowest possible amount of money and achieve the maximum crashing time, as a result both direct and indirect costs will be influenced in the project; therefore, in some researches the terms 'time-cost tradeoff' is also used for this purpose (Nikoomaram *et al*., 2010).

In this article, a company whose managers have decided to schedule a project is chosen for study. Their aim is to minimize the total direct and indirect costs. In practice, simple methods like trial and error can be used for scheduling of projects with few activities. However, in a case which the number of activities is great, scientific methods should be used for scheduling. There are not many computer programs designed for Time-cost trade off. Furthermore, these programs solve the problem generally and might not cover the whole conditions of our problem like available constraints or different objective function. Therefore, if mathematical models are formulated precisely, they can represent the problem under study more realistically and they can be solved using optimization programs.

Rest of this paper is organized as follows: section II gives a review of literature, section III provides some key definitions used in time-cost trade off, section IV includes research methodology. Finally, conclusions are presented in section V.

2. Literature Review:

Extensive investigations have been done about time-cost trade off problems. Some researchers used heuristic algorithms for shortening duration of a project (Siemens, 1971). In many researches, programming models are developed to solve optimally the trade off among time, cost and quality. Linear programming

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(Cusack, 1985; Babu and Suresh, 1996) and dynamic programming (Demeulemeester *et al*., 1996) are examples of these models. In some studies, goal programming models are presented to crash project. In these models, multiple goals are considered, such as meeting a compressed project completion schedule; ensuring that certain activities are not compressed due to overriding quality considerations; confining the expenses to a specified budget; and minimizing the total direct cost of crashing (Vrat and Kriengkrairut, 1986) and (Azaron, 2005; Radasch and Kwak, 1998) included multiple conflicting goals. Several papers developed a solution method considering additional realistic project characteristics such as generalized activity precedence and external time constraint for particular activities (Sakellaropoulos and Chassiakos, 2004). A group of researchers focused their attention on uncertain environment. They recognized the stochastic nature of duration and expenditures and proposed stochastic model for crashing projects which can provide more reliable schedule (Tavares, 1994; Klerides and Hadjiconstantinou, 2010; Ke *et al*., 2009). In many projects, required information for estimation of project parameters is not available or is incomplete. Also in many cases the project is under performance for the first time, this compels us to use expert opinion in forecasting the project parameters. Some authors have claimed that fuzzy set theory is more appropriate to model these problems (Ghazanfari *et al*., 2007). Among them is Sen's research (Sen *et al*., 2001) in which Fuzzy set theory is used to model the uncertainties of activity durations.

3. Key Concepts:

In time-cost trade off analysis, project managers generally aim to schedule a project completion time in which total costs of project are in minimum. This goal can change a little in different models of this problem. For instance, if there is a supposed budget in a project, the goal is to find the greatest reduction in project completion time using this specified budget. Duration of an activity can be decreased hiring more workers or using extra resources. However, it is obvious that duration of an activity can not be shortened to any desired value. Furthermore, for some activities there is no possibility to be crashed even by using more equipment because of their nature. For executing an activity in a project, it is required to assign some resources which are directly engaged in this activity. Costs required for providing these resources are known as "direct costs". For executing a project, beside the resources which are directly assigned to activities, other affairs such as management, monitoring, engineering, and accounting are required that fulfilling of these affairs needs budget. Costs required for performing aforementioned affairs are defined as "indirect costs" which are ordinarily computed per unit of schedule time in a project.

In the subject of time-cost trade off problems, duration required for execution of activities can be classified into normal and crash times which are defined as follows:

Normal Activity Time (D_n):

It is the shortest time which an activity can be executed with the minimum direct costs. In Figure 1, the minimum direct cost is point "C_n" and this hint should be considered for all durations greater than "D_n" the direct cost is in minimum but the shortest duration i.e. "D_n" is defined as normal time.

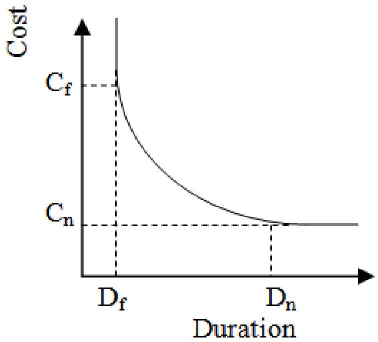

Fig. 1: Time-cost trade off curve.

Crash Activity Time (D_f):

It is the shortest time which execution of an activity is possible.

Normal Activity Cost (C_n):

Total of the direct costs when an activity is executed in its normal time is called normal activity cost.

Crash Activity Cost (C_f):

Total of the direct costs when an activity is executed in its crash time is called crash activity cost.

Activity Cost Slope (C):

It is an amount of extra direct costs which should be paid in return for one unit of time reduction in duration of activity execution. For the cases with non-linear relationship between duration and direct costs like in Figure 1, the cost slope is not identical for different durations and differential should be used to compute cost slope in each point. But, equation (1) can be utilized to calculate cost slope for linear curve.

$$C = \left| \frac{C_f - C_n}{D_f - D_n} \right| \quad (1)$$

4. Research Methodology:

4-1. Brief Description of the Company:

The company considered for the study is a workshop which produces industrial machinery according to customer offer. The productions of this company are mainly used in food industry. Therefore, quality assurance is an important factor in controlling the activities of this company. In addition to quality, time is the next main aspect as customers want the project to be completed in an appointed time. Since orders accepted by the company have the nature of a new project, planning department aims to control time through project management techniques and methods. The primary objective of this department for scheduling each project is to minimize the total of direct and indirect costs of project. Afterwards, selling department takes the necessary actions to contract with customer according to time scheduled by the planning department.

4-2. Gathering the Data for an Accepted Order:

The company has accepted to make an order which is a subassembly in a specific machine for making aluminum foil container. Upon accepting an order by company, manufacturing department prepares a part list for this order at first. This subassembly is defined with code '09' and each of the parts forming this subassembly is named with '09XX' which 'XX' is a cardinal number. Then, this department makes ready a route sheet for each part. Planning department develops a list of activities for project using defined processes in each route sheet. Based on the available and required resources for each process, manufacturing department calculates both a normal time and a least possible time for each activity which are used as normal and crash time by planning department. Various processes on lathe, milling, shaping, and drilling machines are performed in order to manufacture a part as well as heat treatment. These processes should be done in a special order which is specified in route sheet for each part. Theses orders are used to specify precedence relationships between activities of the project. A machining process should be completed in order that the next process can be started; therefore, precedence relationships are considered as 'Finish to Start'. Planning department with cooperation of accounting department estimates normal and crash cost for each activity based on the required resources for performing activity in normal and crash time. Furthermore, accounting department determines indirect cost as currency unit per each time unit. Time unit in this project is day and currency unit is rial which is approximately 1/10000 of US dollar. Cost slope is calculated using equation 1. Table 1 summarizes the necessary data for this project.

4-3. Mathematical Model for Determining the Optimum Time:

In this study, first necessary data are collected; then, a suitable mathematical model is chosen based on the available pre-assumptions, constraints, and objective criterion of the problem. These factors are explained briefly in the rest of this section.

Pre-assumptions in this study are as follows:

- Duration for each activity of the project is deterministic; probabilistic time is not considered.
- Project activities are deterministic. This implied that all scheduled activities should be executed in the future.
- All precedence relationships between activities are 'Finish to Start Type'. In other words, a successor activity can start when a predecessor activity finishes.
- The relationship between time and cost is linear.
- The objective criterion in this study is to find an optimum time for project completion where total cost of project is in minimum. Objective function can be computed by equation 2.

Table 1: The input data for the project.

Activity name	Activity Description	Predecessor	Time		Direct cost		Cost slope
			Normal	Crash	Normal	Crash	
A	Milling of 0901	-	15	13	225000	265000	20000
B	Grinding of 0901	A	2	1	26000	28000	2000
C	Milling of 0902	-	14	10	210000	290000	20000
D	Grinding of 0902	C	2	1	26000	28000	2000
E	Milling of 0903	-	2	1	30000	50000	20000
F	Turning of 0903	E	2	1	16000	17000	1000
G	Grinding of 0903	F	4	3	52000	54000	2000
H	Milling of 0904	-	3	1	45000	85000	20000
I	Turning of 0904	H	2	1	16000	17000	1000
J	Grinding of 0904	I	5	3	65000	69000	2000
K	Milling of 0905	-	3	2	45000	65000	20000
L	Turning of 0905	k	4	2	32000	34000	1000
M	Grinding of 0905	L	3	1	39000	43000	2000
N	Milling of 0906	-	5	2	75000	135000	20000
O	Grinding of 0906	N	6	4	78000	82000	2000
P	Milling of 0907	-	5	3	75000	105000	15000
Q	Grinding of 0907	P	7	5	91000	95000	2000
R	Milling of 0908	-	2	1	20000	35000	15000
S	Milling of 0909	-	5	3	50000	80000	15000
T	Turning of 0910	-	3	1	24000	26000	1000
U	Milling of 0910	T	2	1	20000	35000	15000
V	Milling of 0911	-	12	9	180000	240000	20000
W	Grinding of 0911	V	8	6	104000	108000	2000
Total		-	-	-	1544000	1986000	-

Indirect cost per each unit of time is 25000 rial.

$$Min\,Z = H(t_n - t_1) + k_n + \sum_i \sum_j C_{ij}(D_{n(ij)} - d_{ij}) \quad (2)$$

Where:

'H' is indirect cost per each unit of time and 't_i' is planned date for event 'i'. Project completion time can be obtained by (t_n- t_1); therefore, total indirect cost of project is equal to first term in the above equation, i.e. 'HÍ(t_n- t_1)'.

'K_n' is the sum of direct costs, when activities are executed in their normal time. When duration of activity 'i-j' is shortened from its normal time '$D_{n(ij)}$' to planned duration 'd_{ij}', additional direct cost for this acceleration is equal to '$C_{ij}(D_{n(ij)}$-d_{ij}). As a result, sum of the two last terms in equation 2 shows the total direct cost of a project.

There are two types of constraint in this problem as follows:

- Precedence constraints: Finish date of head event for each activity is greater than or equal to start date of its tail event plus activity duration. This constraint can be formulated by equation 3 for each activity like i-j.

$$t_j \geq ti + d_{ij} \Rightarrow t_j - t_i \geq di_j \quad (3)$$

Figure 2 represents the precedence graph for the problem under study. For instance, this equation for activity 'B' can be written in the following form:

$$t_{16} - t_2 \geq d_B$$

For critical activities, the constraint will be in the form of equality and for non-critical activities in the form of inequality. Left hand side of this constraint will be equal to slack of activity.

Practical duration constraint: Planned duration of each activity should be between its normal and crash time. This constraint can be formulated by equation 4 for each activity like i-j.

$$D_{f(ij)} \leq d_{ij} \leq D_{n(ij)} \quad (4)$$

4-4. Developing the Mathematical Model in LINGO:

In this section, mathematical model of problem under study is programmed in LINGO, which is a software program used for solving operation research problems, according to the explanations of the previous section (see figure 3). This is a linear problem whose variables are event dates 'T_i' and planned duration 'd_{ij}' for each activity.

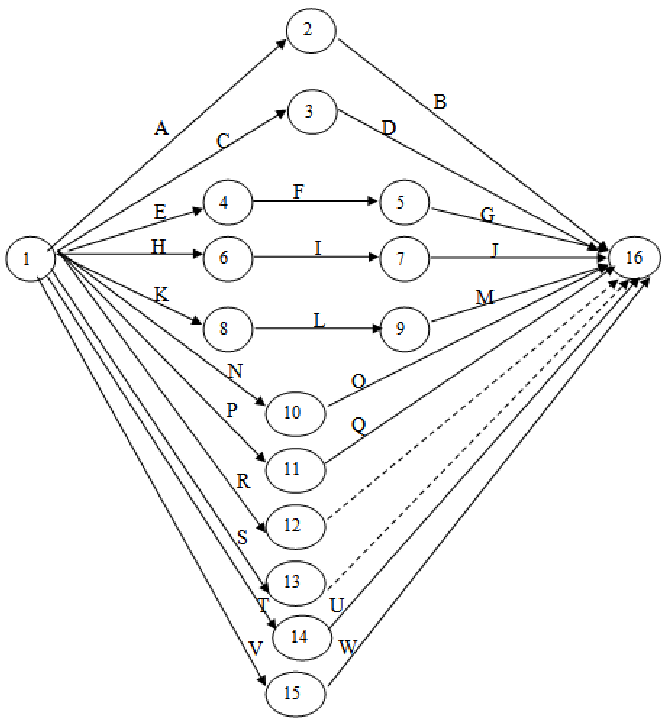

Fig. 2: Precedence graph for the problem under study.

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MODEL:

!Objective Function: Here is the total of direct and indirect costs;
Min  = 25000*(t16-t1)+20000*(15-dA)+2000*(2-dB)+2000*(14-dC)
+2000*(2-dD)+20000*(2-dE)+1000*(2-dF)+2000*(4-dG)+20000*(3-dH)
+1000*(2-dI)+2000*(5-dJ)+20000*(3-dK)+1000*(4-dL)+2000*(3-dM)
+20000*(5-dN)+2000*(6-dO)+15000*(5-dP)+2000*(7-dQ)+15000*(2-dR)
+15000*(5-dS)+1000*(3-dT)+15000*(2-dU)+20000*(12-dV)+2000*(8-dW)+1544000;


! Constraint 1: Finish date of head event for each activity is greater than
or equal to the sum of activity duration  and its tail event's start date;
t2-t1>=dA;     t16-t2>=dB;  t3-t1>=dC;   t16-t3>=dD;   t4-t1>=dE;
t5-t4>=dF;     t16-t5>=dG;  t6-t1>=dH;   t7-t6>=dI;    t16-t7>=dJ;
t8-t1>=dK;     t9-t8>=dL;   t16-t9>=dM;  t10-t1>=dN;   t16-t10>=dO;
t11-t1>=dP;    t16-t11>=dQ; t12-t1>=dR;  t16-t12>=0;   t13-t1>=dS;
t16-t13>=0;    t14-t1>=dT;  t16-t14>=dU;  t15-t1>=dV;  t16-t15>=dW;

! Constraint 2: Duration of each activity is between normal and crash time;
dA>=13; dA<=15;  dB>=1;  dB<=2;  dC>=10; dC<=14; dD>=1;  dD<=2;
dE>=1;  dE<=2;   dF>=1;  dF<=2;  dG>=3;  dG<=4;  dH>=1;  dH<=3;
dI>=1;  dI<=2;   dJ>=3;  dJ<=5;  dK>=2;  dK<=3;  dL>=2;  dL<=4;
dM>=1;  dM<=3;   dN>=2;  dN<=5;  dO>=4;  dO<=6;  dP>=3;  dP<=5;
dQ>=5;  dQ<=7;   dR>=1;  dR<=2;  dS>=3;  dS<=5;  dT>=1;  dT<=3;
dU>=1;  dU<=2;   dV>=9;  dV<=12; dW>=6;  dW<=8;

END
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Fig. 3: Mathematical model of problem in LINGO.

Optimum solution is achieved by software and other required data can be computed as the following descriptions:

- Project completion time 'T' is equal to ending event date, i.e. t_{16} minus starting event date in network i.e. t_1.
- Start date for each activity is equal to date of its tail event.
- Finish date for each activity is equal to date of its head event.
- Planned duration for each activity (d_{ij}) is directly obtained from solving the model.
- Slack for each activity can be computed by equation 5.

$$Slack = Finish\,date - Start\,date - Planned\,duration \tag{5}$$

Activities with slack of zero are considered as critical activities meaning that delaying or increasing duration of these activities can lead to postpone project completion time.

Total direct cost of each activity can be computed by adding normal activity cost and extra cost needed for shortening the activity duration. For linear cost slope curve, equation 6 can be utilized to calculate this value.

$$Total\,direct\cos t = C_{n(ij)} + C_{ij} \times (D_{n(ij)} - d_{ij}) \tag{6}$$

Where '$C_{n(ij)}$' is activity normal cost, 'C_{ij}' is cost slope, 'D_n' is normal activity cost and 'd_{ij}' is planned duration for activity i-j.

Indirect costs are computed for total project not for every single activity separately. Total indirect cost of project is product of indirect cost of project per each unit of time 'H' and project completion time 'T' (see equation 7).

$$Total\,indirect\cos t = H \times T = H \times (t_n - t_1) \tag{7}$$

Table 2 exhibits the summary of aforementioned explanations.

Table 2: Schedule table of project.

Activity name	Activity Description	Tail Event (ti)	Head Event (ti)	Start Date	Planned Duration (dij)	Finish Date	Slack	Critical	Direct Cost
A	Milling of 0901	T1	T2	0	15	15	0	yes	225000
B	Grinding of 0901	T2	T16	15	1	16	0	yes	28000
C	Milling of 0902	T1	T3	0	14	14	0	yes	210000
D	Grinding of 0902	T3	T16	14	2	16	0	yes	26000
E	Milling of 0903	T1	T4	0	2	2	0	yes	30000
F	Turning of 0903	T4	T5	2	2	4	0	yes	16000
G	Grinding of 0903	T5	T16	4	4	16	8	no	52000
H	Milling of 0904	T1	T6	0	3	3	0	yes	45000
I	Turning of 0904	T6	T7	3	2	5	0	yes	16000
J	Grinding of 0904	T7	T16	5	5	16	6	no	65000
K	Milling of 0905	T1	T8	0	3	3	0	yes	45000
L	Turning of 0905	T8	T9	3	4	7	0	yes	32000
M	Grinding of 0905	T9	T16	7	3	16	6	no	39000
N	Milling of 0906	T1	T10	0	5	5	0	yes	75000
O	Grinding of 0906	T10	T16	5	6	16	5	no	78000
P	Milling of 0907	T1	T11	0	5	5	0	yes	75000
Q	Grinding of 0907	T11	T16	5	7	16	4	no	91000
R	Milling of 0908	T1	T12	0	2	2	0	yes	20000
S	Milling of 0909	T1	T13	0	5	5	0	yes	50000
T	Turning of 0910	T1	T14	0	3	3	0	yes	24000
U	Milling of 0910	T14	T16	3	2	16	11	no	20000
V	Milling of 0911	T1	T15	0	10	12	2	no	220000
W	Grinding of 0911	T15	T16	12	6	16	-2	no	108000
						TOTAL DIRECT COST			1590000
						INDIRECT COST			400000
						TOTAL COST			1990000

5. Conclusions:

In case all activities are executed in their crash time, the project will be completed in the shortest possible duration. Therefore, indirect cost of project will be in minimum, but instead direct costs of project will be in the greatest possible value. In comparison, when activities are executed in their normal time, the project completion time will be in the longest duration; therefore, indirect cost of project will be in minimum and direct cost of project in maximum. To sum up, by decreasing one cost, the other increases. With application of a suitable mathematical model, an optimum completion time will be computed for project in which total costs of project, i.e. sum of indirect and direct cost will be in the least possible value. Table 3 indicates project costs for these completion times.

Table 3: Project costs for different completion times

Parameter Case	Completion Time	Direct Cost	Indirect Cost	Total Cost
Normal	20	1544000	500000	2044000
Crash	15	1986000	375000	2361000
Optimum	16	1590000	400000	1990000

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