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Optimal Clustering of GPS Satellites Set using Modified ACO Algorithm

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Abstract: The Geometric Dilution of Precision (GDOP) is an important factor that demonstrates the effectiveness of the GPS satellite geometry on the positional navigation performance. The matrix inversion conventional method for GPS GDOP clustering has a large amount of operation, which would be a burden for real-time application. In this paper a new approach based on Modified Ant Colony Optimization (MACO) algorithm for satellites geometry clustering is proposed. The proposed method applies different way for update pheromone which uses information of local and global searches. The MACO-based approach is capable of evaluating all subsets of satellites and hence reduces the computational burden. Simulation results indicate the proposed method has more performance to converge upon the optimal value in the GPS GDOP clustering.

Key words: Clustering, GPS GDOP, Modified Ant Colony Optimization

INTRODUCTION

The GPS navigation performance is dependent on the GPS satellite geometry with respect to the GPS antenna. The Geometric Dilution of Precision (GDOP) is the value that indicates the effectiveness of the GPS satellite geometry on the navigation performance. The well-established relation between GDOP and the navigation solution accuracy is $\sigma = GDOP.\sigma_\rho$, where σ_ρ is the standard deviation of the undifferenced range measurement error and $\sigma = \sqrt{\sigma_H^2 + \sigma_V^2 + \sigma_T^2}$, where σ_H, σ_V and σ_T are standard deviations of the horizontal position, vertical position and time solution errors, respectively (Parkinson, 1996).

The most GPS receivers track only some of the GPS satellites in the Field of View (FOV). Even when a receiver tracks all GPS satellites in the FOV, only a subset of the satellites is used for the navigation solution (Simon and El-Sherief, 1995). The DOP concept plays an important role in the algorithm to select a subset of GPS satellites to track or to be used for the navigation solution. By checking the values of the DOPs, a receiver can determine the most appropriate set of GPS satellites for its purpose (Yoon and Lundberg, 2001).

The most straightforward approach for obtaining GPS GDOP is to use matrix inversion to all combinations and select the minimum one. However, the matrix inversion by computer presents a computational burden to the navigation computer. For the case of processing four satellite signals, it has been shown that GPS GDOP is approximately inversely proportional to the volume of the tetrahedron formed by four satellites. Therefore, it is optimum to select satellite such that the volume is as large as possible, which is sometimes called the maximum volume method. However, it is not universal acceptable, since it does not guarantee optimum selection of satellites (Jwo and Lai, 2007; Wu *et al.,* 2011).

Clustering is the unsupervised classification of patterns (observations, data items or feature vectors) into clusters. The clustering problem has been addressed in many contexts and by researchers in many disciplines; this reflects its broad appeal and usefulness as one of the steps in exploratory data analysis. However, clustering is a difficult problem combinatorial and differences in assumptions and contexts in different communities have made the transfer of useful generic concepts and methodologies slow to occur (Jain *et al.,* 1999).

Ant Colony Optimization (ACO) is a population-based metaheuristic that can be used to find approximate solutions to difficult optimization problems (Dorigo and Gambardella, 1997). In ACO, a set of software agents called “artificial ants” find the shortest route between a food source and their nest. The ants communicate with

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one another by means of pheromone trails and exchange information about which path should be followed. The more the number of ants traces a given path, the more attractive this path (trail) becomes and is followed by other ants by depositing their own pheromone. This auto catalytic and collective behavior results in the establishment of the shortest route. This real-life search behavior was the key motivation factor leading to the formulation of artificial ant algorithms to solve several large-scale combinatorial and function optimization problems. In all these algorithms, a set of ant-like agents or software ants solve the problem under consideration through a cooperative effort. This effort is mediated by exchanging information on the problem structure the agents concurrently collect while stochastically building solutions (Shelokar *et al.,* 2004).

In this research, a new approach based on Modified ACO (MACO) algorithm for GPS GDOP clustering is proposed. This paper is organized as follow. Section II introduces the GPS GDOP calculation using matrix inversion conventional method. Standard-ACO and MACO algorithms are described in section III. Section IV proposes MACO algorithm developed for GPS GDOP clustering. Section V, the experimental results and discussions are reported with data. Conclusions are presented in section VI.

GPS GDOP Calculation Using Matrix Inversion Conventional Method:

On an unknown location *u* for the user, the pseudo-range observation equation can be approximately shown as:

$$\rho_u^j = \sqrt{(x_j - x_u)^2 + (y_j - y_u)^2 + (z_j - z_u)^2} + b_u - c\Delta t_j + d_{iono} + d_{trop} + v_u^j \tag{1}$$

where ρ_u^j is the pseudo-range measurements between the satellite *j* and the user location *u*; (x_j, y_j, z_j) is the coordinate values of the satellites *j* in the earth center coordinate; (x_u, y_u, z_u) is the coordinate of *u*; b_u denotes the unknown offset of the user's clock relative to GPS system time, which unit is meter; Δt_j denotes the unknown offset of the satellite's clock relative to GPS system time; *c* represents the transmit speed of the electromagnetic wave in the air; d_{iono} and d_{trop} are the delay due to ionosphere and the delay due to troposphere, respectively (Yong and Lingjuan, 2004).

Suppose that the receiver's approximate location is (x_u^0, y_u^0, z_u^0) and its corresponding baseline error is $(\Delta x, \Delta y, \Delta z)$. Then the equation (1) can be rewritten as a linear equation:

$$v_u^j = e_x^j \Delta x + e_y^j \Delta y + e_z^j \Delta z - b_u + l_u^j \tag{2}$$

where (e_x^j, e_y^j, e_z^j) is the unit vector from *u* to *j* and are computed as:

$$e_x^j = \frac{x_j - x_u^0}{\hat{\rho}_u^j},\ e_y^j = \frac{y_j - y_u^0}{\hat{\rho}_u^j},\ e_z^j = \frac{z_j - z_u^0}{\hat{\rho}_u^j};\ \hat{\rho}_u^j = \sqrt{(x_j - x_u^0)^2 + (y_j - y_u^0)^2 + (z_j - z_u^0)^2} \tag{3}$$

where

$$l_u^j = \rho_u^j - \hat{\rho}_u^j + c\Delta t_j - d_{iono} - d_{trop} \tag{4}$$

In the equation (4), each term can be acquired from satellite ephemeris and approximate coordinate, so we look them as the known items. Suppose there are *n* satellites ($n \geq 4$) at an observation epoch, then the equation (2) takes the form:

$$V = AX + L \tag{5}$$

where $X = [\Delta x\ \Delta y\ \Delta z\ -b_u]^T$ just is the indeterminate parameter; and

$$L=[l_u^1 \;\; l_u^2 \;\; \cdots \;\; l_u^n]^T \;,\; V=[v_u^1 \;\; v_u^2 \;\; \cdots \;\; v_u^n]^T \;,\; A=\begin{bmatrix} e_x^1 & e_y^1 & e_z^1 & 1 \\ e_x^2 & e_y^2 & e_z^2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ e_x^n & e_y^n & e_z^n & 1 \end{bmatrix} \tag{6}$$

According to the algorithm of Least Square (LS), the indeterminate parameter is given by:

$$\hat{X}=(A^TA)^{-1}A^TL \tag{7}$$

Then the error of GPS positioning is:

$$Var(\Delta x)=E[(\hat{X}-X)(\hat{X}-X)^T]=E(V^TV).(A^TA)^{-1} \tag{8}$$

Assume that each observation standard deviation is σ. The equation (8) also can use the following expression:

$$Var(\Delta x)=\sigma^2(A^TA)^{-1}=\sigma^2\begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix} \tag{9}$$

where the diagonal elements are the variances of the unknown parameters (Phatak, 2001). It is obvious that the precise of positioning is deeply influenced by diagonal elements. So GPS GDOP can be defined as:

$$GPS\,GDOP=\sqrt{tr(A^TA)^{-1}}=\sqrt{g_{11}+g_{22}+g_{33}+g_{44}} \tag{10}$$

where *tr*(.) is the symbol of matrix trace computing.

Standard ACO Algorithm:

Ants start with empty solution strings and in the first iteration the elements of the pheromone matrix are initialized to the same values. With the progress of iterations, the pheromone matrix is updated depending upon the quality of solutions produced. Each agent used in algorithm is defined with solution strings expressed with *S* of length *N* and at start of the algorithm solution string of each element is empty. Each element of string corresponds to one of the test samples and its value includes which cluster it will be assigned. To construct a solution, the agent uses the pheromone trail information to allocate each element of string *S* to an appropriate cluster label. The trail value, τ_{ij} at location (*i,j*) represents the pheromone concentration of sample *i* associated to the cluster *j*. So, for the problem of separating *N* samples into *K* clusters the size pheromone matrix is *N*×*K*. Thus, each sample is associated with *K* pheromone concentrations. The pheromone trail matrix evolves as we iterate. At any iteration level, each agent will develop solutions showing the probability of each ant belonging to which cluster using this pheromone matrix (Shelokar *et al.,* 2004).

After generating the solutions of *R* agents, a local search is performed to further improve fitness of these solutions. The pheromone matrix is then updated depending on the quality of solutions produces by the agents. Then, the agents build improved solutions depending on the pheromone matrix and the above steps are repeated for certain number of iterations. At the end of any iteration level each agent generates the solution using the information derived from updated pheromone matrix. Each agent selects a cluster number with a probability value for each element of *S* string to form its own solution string *S*. The quality of constructed solution string *S* is measured in terms of the value of objective function for a given data-clustering problem. This objective function is defined as the sum of squared Euclidian distances between each object and the center of belonging cluster. Then, the elements of the population, namely agents are sorted increasingly by the objective function values. Because, the lower objective function value, the higher fitness to the real solution, namely, lower objective function values are more approximated to real solution values. Most of existing ACO algorithms uses some local search procedures to develop the generated solutions discovered by agents. Local search helps to

generate better solutions, if the heuristic information can not be discovered easily. Local search is applied on all generated solutions or on a few percent *R*. These solutions can be accepted only if there is an improvement on the fitness, namely, if the newly computed objective function value is lower than the first computed value, newly generated solution replaces the old one.

After the local search procedure, the pheromone trail matrix is updated. Such a pheromone updating process reflects the usefulness of dynamic information provided by agents. The pheromone matrix used in ACO algorithm is a kind of adaptive memory that contains information provided by the previously found superior solutions and is updated at the end of the iteration. The pheromone updating process used includes best *L* solutions discovered by *R* agents at iteration level *t*. This *L* agent mimics the real ants' pheromone deposition by assigning the values of solutions (Kekec *et al.,* 2006).

An optimal solution is that solution which minimizes the objective function value. If the value of best solution in memory is updated with the best solution value of the current iteration if it has a lower objective function value than that of the best solution in memory, otherwise the best solution in memory kept. This process explains that an iteration of the algorithm is finished. Algorithm iterates these steps repeatedly until a certain number of iterations and solution having lowest function value represents the optimal partitioning of objects of a given dataset into several groups.

Steps of ACO algorithm are summarized as follows (Shelokar *et al.,* 2004; Dorigo and Blum, 2005):

Step1:

Generation of new *R* solutions by software ants using the modified pheromone trail information available from previous iteration. To generate a solution *S*, the agent selects cluster number for each element of string *S* by one of the following ways:

(i) Using probability q_0, cluster having the maximum pheromone concentration is chosen (q_0 being a priori defined number, $0 < q_0 < 1$, for the illustrative example and in our simulations), and/or

(ii) One of the *K* clusters using a stochastic distribution with a probability (1-q_0), denoted as P_{ij}.

The first process is known as exploitation whereas the latter is termed as biased exploration. The second process chooses any one of the *K* clusters with a normalized pheromone probability (pheromone probability normalized to 1) given by:

$$p_{ij} = \frac{\tau_{ij}}{\sum_{k=1}^{k=K} \tau_{ik}} \; ; \; j = 1,...,K \tag{11}$$

where p_{ij} is the normalized pheromone probability for element *i* belongs to cluster *j*. The quality of solution constructed is measured in terms of the value of objective function for a given data-clustering problem. This objective function is defined as the sum of squared Euclidean distances between each object and the center of belonging cluster. Consider a given dataset of *N* objects $\{x_1, x_2, ..., x_N\}$ in R^n -dimensional space to be partitioned into a number, say *K*, of clusters or groups. The mathematical formulation of the data-clustering problem can be described as:

$$Min \;\; F(W,M) = \sum_{j=1}^{j=K} \sum_{i=1}^{i=N} \sum_{v=1}^{v=n} w_{ij} \left\| x_{iv} - m_{jv} \right\|^2 \tag{12}$$

Such that:

$$\sum_{j=1}^{j=K} w_{ij} = 1 \; ; \; i = 1, ..., N \tag{13}$$

$$\sum_{i=1}^{i=N} w_{ij} \geq 1 \; ; \; j = 1, ..., K \tag{14}$$

where x_{iv} is a value of *v*-th attribute of *i*-th sample, *M* a cluster center matrix of size $K \times n$, m_{iv} an average of the *v*-th attribute values of all samples in the cluster *j*, *W* a weight matrix of size $N \times K$, w_{ij} an associated weight of object x_i with cluster *j* which can be assigned as:

$$w_{ij} = \begin{cases} 1 \ if \ \ object \ \ i \ \ is \ \ contained \ \ in \ \ cluster \ \ j \\ 0 \ \ otherwise \end{cases} ; i = 1,...,N \ , \ j = 1,...,K \tag{15}$$

After getting w_{ij}'s, the center of each cluster, m_{jv} can be obtained as:

$$m_{jv} = \frac{\sum_{i=1}^{i=N} w_{ij} x_{iv}}{\sum_{i=1}^{i=N} w_{ij}} \ ; \ j = 1,\cdots,K \ , \ v = 1,\cdots,n \tag{16}$$

Step2:

Performing local search operation on the newly generated solutions. The local search algorithm can be written as follows:

With local search probability threshold *Pls* in [0,1], a neighbor of S_k, k = 1,...,L is generated as:

(i) k = 1.

(ii) Let S_t be a temporary solution and assign $S_t(i) = S_k(i), i = 1,...,N$.

(iii) For each element i of S_t, draw a random number r in (0,1). If $r \le p_{ls}$, an integer j in the range (1,K), such that $S_k(i) \ne j$ is randomly selected and let $S_t(i) = j$.

(iv) Calculate cluster centers and weights associated with solution string S_t and find its objective function value using equation (12) as F_t. If F_t is less than F_k, then $S_k = S_t$ and $F_k = F_t$.

(v) $k = k$+1; if $k \le L$ go to step (ii), else stop.

Step3:

Updating pheromone trail matrix. The trail information is updated using the following rule as:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \sum_{l=1}^{l=L} \Delta\tau_{ij}^{l} \ ; \ i = 1,\cdots,N \ , \ j = 1,\cdots,K \tag{17}$$

where ρ is the persistence of trail that lies between [0,1] and (1-ρ) the evaporation rate. Higher value of ρ suggests that the information gathered in the past iterations is forgotten faster. The amount $\Delta\tau_{ij}^{l}$ is equal to $\frac{1}{F_l}$, if cluster j is assigned to i-th element of the solution constructed by ant l and zero otherwise. An optimal solution is that solution which minimizes the objective function value. The value of best solution in memory is updated with the value of the solution obtained as "current iteration best solution" if it is having a lower objective function value than that of the best solution in memory. The algorithm repeatedly carries out these three steps for a maximum number of given iterations, and solution having lowest function value represents the optimal partitioning of objects of a given dataset into several groups.

In this paper for improving data-clustering performance with ACO algorithm, we modify update pheromone step. In this method we use both global search information and local search information for update pheromone because in cases that local search didn't perform the best solution (when calculated fitness function is more than best fitness function), the use of best solution with the global fitness function can be the better effect in data-clustering. In cases that local search performs the best solution (when calculated fitness function is less than best fitness function), local and global search information is equal. In update pheromone, we use both of local and global information with constant factor that this constant factor is the result of the try and test. The modified update pheromone is using the following rule as:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \frac{A}{F_l} + \frac{B}{F_{best}} \ ; \ i = 1,\cdots,N \ , \ j = 1,\cdots,K \tag{18}$$

If cluster j is assigned to i-th element of the solution constructed by ant l and zero otherwise and/or if cluster j is assigned to i-th element of the best solution constructed by ant l and zero otherwise.

Mapping System for Geometry Matrix:

The geometry matrix A is given by (Parkinson, 1996):

$$A=\begin{bmatrix} Cos(E1)*Sin(Az1) & Cos(E1)*Cos(Az1) & Sin(E1) & 1 \\ Cos(E2)*Sin(Az2) & Cos(E2)*Cos(Az2) & Sin(E2) & 1 \\ Cos(E3)*Sin(Az3) & Cos(E3)*Cos(Az3) & Sin(E3) & 1 \\ Cos(E4)*Sin(Az4) & Cos(E4)*Cos(Az4) & Sin(E4) & 1 \end{bmatrix} \tag{19}$$

where E and Az are elevation and azimuth of satellite, respectively. Since A^TA is a 4×4 matrix, it has four eigenvalues, $\lambda_i (i=1,2,3,4)$. The mapping is performed by defining the four variables:

$$x_1=\lambda_1+\lambda_2+\lambda_3+\lambda_4=trace(A^TA) \tag{20}$$

$$x_2=\lambda_1^2+\lambda_2^2+\lambda_3^2+\lambda_4^2=trace[(A^TA)^2] \tag{21}$$

$$x_3=\lambda_1^3+\lambda_2^3+\lambda_3^3+\lambda_4^3=trace[(A^TA)^3] \tag{22}$$

$$x_4=\lambda_1\lambda_2\lambda_3\lambda_4=\det(A^TA) \tag{23}$$

Therefore each object is defined by four attributes, i.e., n = 4.

Experimental Results:

We implemented the proposed MACO algorithm for clustering on dataset of GPS GDOP. The algorithm was executed in "Matlab 7.6" language. The data file containing of 100 patterns was used for testing the proposed method. Table I shows rating of three classes in GPS GDOP clustering.

Table 1: rating of three classes in GPS GDOP Clustering

Quality	GDOP Value
Good	1-3
Medium	3-5
Bad	>5

The choice of the algorithm parameters is very important. In this paper, we obtain optimal values for the variables with try and test. Number of clusters, agents, local search agents, iterations and initial pheromone values, evaporation rate of pheromone and some values needed for the algorithm are listed in Table II.

Table 2: The parameters of the proposed MACO

Parameters	Value
K	3
R	10
L	2
τ_0	0.01
ρ	0.04
q_0	0.50
p_{ls}	0.70

A and B coefficients with try and test are equal to 0.4 and 0.9, respectively. Figures (1) and (2) show the clustering using the proposed MACO for 100 dataset of GPS GDOP. The simulations results show that correct clustering percentage of proposed method is more than %95.

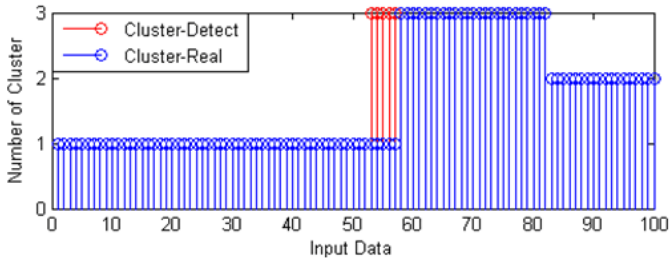

Fig. 1: Clustering using the proposed MACO for 100 GPS GDOP dataset

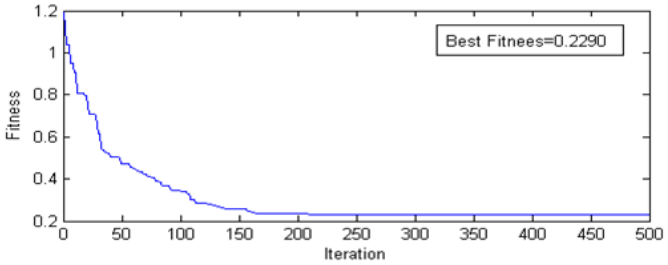

Fig. 2: Fitness function for 100 dataset of GPS GDOP

The reference (Jwo and Lai, 2007) classified GPS GDOP using four type neural networks. The proposed method in our paper using ACO has better performance than the proposed models in reference (Jwo and Lai, 2007), since error of classification using the proposed MACO model and also CPU time of this algorithm are lower than other models.

Conclusions:

The GDOP has been widely used as an accuracy metric for navigation and tracking systems. Since high accuracy in a positioning system requires both accurate measurement of the range and a good geometric relationship between the mobile device and the measuring points, the analysis of GDOP is an essential feature in determining the performance of a positioning system. This paper proposed a new approach based on MACO algorithm for satellites geometry clustering. Comparing with other existing schemes, the simulations results demonstrate that the proposed algorithm can achieve better accuracy and also better CPU time in GPS GDOP clustering.

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