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Classical Estimations of the Exponentiated Gamma Distribution Paremeters with Presence of *K* Outliers

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Abstract: This paper deals with the estimation of parameters of the Exponentiated Gamma (EG) distribution with presence of k outliers. The maximum likelihood and moment of the estimators are derived. These estimators are compared empirically using Monte Carlo simulation when all the parameters are unknown. There bias and MSE are investigated with help of numerical technique.

Key words: *Exponentiated Gamma distribution, Maximum Likelihood Estimator, Moment Estimator, Outliers, Newton-Raphson method, Monte-Carlo simulation.*

INTRODUCTION

Recently a new distribution, called Exponentiated Gamma (EG) distribution, has been introduced. This distribution was introduced by Gupta et al. (1998) which has a probability density function (p.d.f.) of the form

$$f(x;\alpha) = \alpha\, x\, e^{-x}(1-e^{-x}(1+x))^{\alpha-1} \quad ; \quad x>0 \ , \ \ \alpha>0, \tag{1}$$

and a cumulative distribution function (c.d.f.)

$$F(x;\alpha) = (1-e^{-x}(1+x))^{\alpha} \quad ; \quad x>0 \ , \ \ \alpha>0, \tag{2}$$

where α is the shape parameter. It is important to mention that when α=1, the Exponentiated Gamma p.d.f. is that of gamma distribution with shape parameter α=2 and scale parameter β=1, i.e., $G(2,1)$. For more details about this distribution, see Shawky and Bakoban (2008c & 2009). Also, characterization from EG distribution based on record values and Bayesian estimations on the EG distribution discussed by Shawky and Bakoban (2008b,a). The density functions of the Exponentiated Gamma distribution can take different shapes. Figure 1 shows the shape of $f(x;\alpha)$ for different values of α.

Dixit, Moore and Barnett (1996), assume that a set of random variables $(X_1, X_2, ..., X_n)$ represent the distance of an infected sampled plant from a plant from a plot of plants inoculated with a virus. Some of the observations are derived from the airborne dispersal of the spores and are distributed according to the exponential distribution. The other observations out of *n* random variables (say *k*) are present because aphids which are know to be carriers of barley yellow mosaic dwarf virus (BYMDV) have passed the virus into the plants when the aphids feed on the sap. Dixit and Nasiri (2001) considered estimation of parameters of the exponential distribution in the presence of outliers generated from uniform distribution. In this paper, we obtain the maximum likelihood and moment estimators of the parameters of the exponentiated gamma distribution in the presence of *k* outliers generated from exponentiated gamma distribution.

We assume that the random variables $(X_1, X_2, ..., X_n)$ are such that *k* of them are distributed with p.d.f $g(x;\theta)$,

$$g(x;\theta) = \theta\, x\, e^{-x}(1-e^{-x}(1+x))^{\theta-1} \quad ; \quad x>0 \ , \ \ \theta>0 \tag{3}$$

and remaining (*n-k*) random variables are distributed with p.d.f $f(x;\alpha)$ given in (1).

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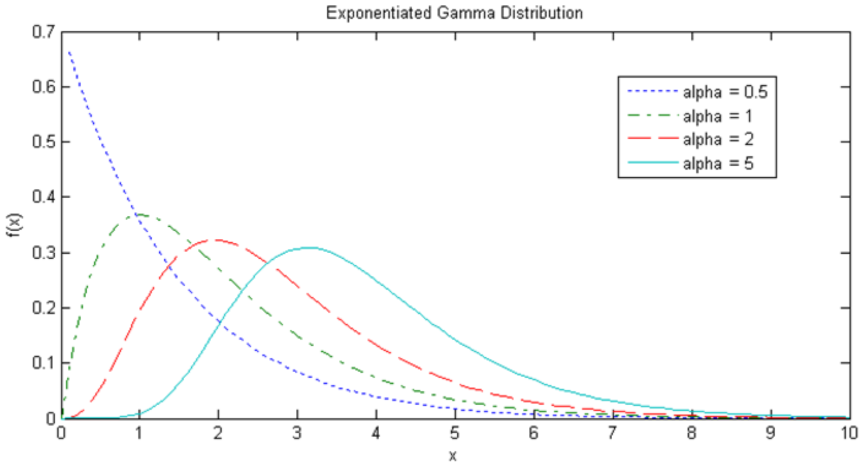

Fig. 1: p.d.f. of EG(α) for different values of α.

The paper is organized as follows: In section 2, we have obtained the joint distribution of $(X_1, X_2, ..., X_n)$ in the presence of k outliers. Section 3 and 4 discusses the methods of moment and maximum likelihood estimators. The different proposed methods have been compared using Monte Carlo simulations and the results have been reported in section 5.

Joint Distribution of $(X_1, X_2, ..., X_n)$ ***with Presence of k Outliers***:

The joint distribution of $(X_1, X_2, ..., X_n)$ in the presence of k outliers can be expressed as

$$f(x_1, x_2, ..., x_n; \alpha, \theta) = \frac{1}{c(n,k)} \prod_{i=1}^{n} f(x_i; \alpha) \cdot \sum_{\underline{A}} \prod_{r=1}^{k} \frac{g(x_{A_r}; \theta)}{f(x_{A_r}; \alpha)} \tag{5}$$

$$= \frac{1}{c(n,k)} \prod_{i=1}^{n} \alpha\, x_i\, e^{-x_i} \left(1 - e^{-x_i}(1+x_i)\right)^{\alpha-1} \cdot \sum_{\underline{A}} \prod_{r=1}^{k} \frac{\theta\, x_{A_r}\, e^{-x_{A_r}} \left(1 - e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-1}}{\alpha\, x_{A_r}\, e^{-x_{A_r}} \left(1 - e^{-x_{A_r}}(1+x_{A_r})\right)^{\alpha-1}}$$

$$= \frac{\alpha^{n-k}\, \theta^{k}}{c(n,k)}\, e^{-\sum_{i=1}^{n} x_i} \prod_{i=1}^{n} \left[x_i \left(1 - e^{-x_i}(1+x_i)\right)^{\alpha-1} \right] \cdot \sum_{\underline{A}} \prod_{r=1}^{k} \left(1 - e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha} \tag{4}$$

where $c(n,k) = n!/((n-k)!k!)$ and $\sum_{\underline{A}} = \sum_{A_1=1}^{n-k+1} \sum_{A_2=A_1+1}^{n-k+2} \cdots \sum_{A_k=A_{k-1}+1}^{n}$. See Dixit (1996), Dixit and Nasiri (2001), and Nasiri and Pazira (2010). From (4), the marginal distribution of X is

$$f(x; \alpha, \theta) = \frac{n-k}{n} \alpha\, x\, e^{-x} \left(1 - e^{-x}(1+x)\right)^{\alpha-1} + \frac{k}{n} \theta\, x\, e^{-x} \left(1 - e^{-x}(1+x)\right)^{\theta-1} \quad ; \quad x > 0. \tag{5}$$

Method of Moment:

The moments can also be obtained in the form of a series which is finite or infinite depending on whether α and θ are integers or not. The raw moments of X may be determined from (5) be direct integration. For $r \in \mathrm{N}$, we find that

$$E(X^r) = \int_0^\infty x^r \left[\frac{n-k}{n} \alpha\, x\, e^{-x} \left(1 - e^{-x}(1+x)\right)^{\alpha-1} + \frac{k}{n} \theta\, x\, e^{-x} \left(1 - e^{-x}(1+x)\right)^{\theta-1} \right] dx$$

$$= \frac{n-k}{n} \int_0^\infty \alpha\, x^{r+1} e^{-x} \left(1 - e^{-x}(1+x)\right)^{\alpha-1} dx + \frac{k}{n} \int_0^\infty \theta\, x^{r+1} e^{-x} \left(1 - e^{-x}(1+x)\right)^{\theta-1} dx. \tag{6}$$

Since $0 < e^{-x}(1+x) < 1$ for x>0 by using the binomial series expansion we have

$$(1-e^{-x}(1+x))^m = \sum_{i=0}^{\infty}\binom{m}{i}(-1)^i e^{-ix}(1+x)^i. \tag{7}$$

Hence,

$$E(X^r) = \frac{(n-k)\alpha}{n}\int_0^{\infty} x^{r+1}e^{-x}\sum_{i=0}^{\infty}\binom{\alpha-1}{i}(-1)^i e^{-ix}(1+x)^i\,dx$$
$$+\frac{k\,\theta}{n}\int_0^{\infty} x^{r+1}e^{-x}\sum_{i=0}^{\infty}\binom{\theta-1}{i}(-1)^i e^{-ix}(1+x)^i\,dx \tag{8}$$

Also, whereas i is integer, by using the binomial series expansion we have

$$(1+x)^i = \sum_{j=0}^{i}\binom{i}{j}x^j$$

Hence,

$$E(X^r) = \frac{(n-k)\alpha}{n}\int_0^{\infty} x^{r+1}e^{-x}\sum_{i=0}^{\infty}\sum_{j=0}^{i}\binom{\alpha-1}{i}\binom{i}{j}(-1)^i e^{-ix}x^j\,dx$$
$$+\frac{k\,\theta}{n}\int_0^{\infty} x^{r+1}e^{-x}\sum_{i=0}^{\infty}\sum_{j=0}^{i}\binom{\theta-1}{i}\binom{i}{j}(-1)^i e^{-ix}x^j\,dx\,. \tag{8}$$

Since the quantity inside the summation is absolutely integrable, interchanging the summation and integration we have

$$E(X^r) = \frac{(n-k)\alpha}{n}\sum_{i=0}^{\infty}\sum_{j=0}^{i}\binom{\alpha-1}{i}\binom{i}{j}(-1)^i\int_0^{\infty}x^{r+j+1}e^{-(i+1)x}\,dx + \frac{k\,\theta}{n}\sum_{i=0}^{\infty}\sum_{j=0}^{i}\binom{\theta-1}{i}\binom{i}{j}(-1)^i\int_0^{\infty}x^{r+j+1}e^{-(i+1)x}\,dx$$
$$= \frac{(n-k)\alpha}{n}\sum_{i=0}^{\infty}\sum_{j=0}^{i}\binom{\alpha-1}{i}\binom{i}{j}(-1)^i\frac{\Gamma(r+j+2)}{(i+1)^{(r+j+2)}} + \frac{k\,\theta}{n}\sum_{i=0}^{\infty}\sum_{j=0}^{i}\binom{\theta-1}{i}\binom{i}{j}(-1)^i\frac{\Gamma(r+j+2)}{(i+1)^{(r+j+2)}}$$
$$= \sum_{i=0}^{\infty}\sum_{j=0}^{i}\binom{i}{j}(-1)^i\frac{(r+j+1)!}{(i+1)^{(r+j+2)}}\left[\frac{(n-k)\,\alpha}{n}\binom{\alpha-1}{i} + \frac{k\,\theta}{n}\binom{\theta-1}{i}\right]. \tag{9}$$

For $\alpha = \theta = \beta$ in case of no outlier presence (k=0), from (9) we get

$$E(X^r) = \sum_{i=0}^{\infty}\sum_{j=0}^{i}\binom{\beta-1}{i}\binom{i}{j}(-1)^i\frac{\beta(r+j+1)!}{(i+1)^{(r+j+2)}}\,, \tag{10}$$

it is proposed by Gupta et al. (1998), also for k=1 it is given by Shadrokh and Pazira (2010). We observe that the infinite series is summable. For r = 1, 2 and 3, $E(X^r)$ is given by

$$E(X) = \sum_{i=0}^{\infty}\sum_{j=0}^{i}\binom{i}{j}(-1)^i\frac{(j+2)!}{(i+1)^{(j+3)}}\left[\frac{(n-k)\,\alpha}{n}\binom{\alpha-1}{i} + \frac{k\,\theta}{n}\binom{\theta-1}{i}\right] \tag{11}$$

$$E(X^2) = \sum_{i=0}^{\infty}\sum_{j=0}^{i}\binom{i}{j}(-1)^i\frac{(j+3)!}{(i+1)^{(j+4)}}\left[\frac{(n-k)\,\alpha}{n}\binom{\alpha-1}{i} + \frac{k\,\theta}{n}\binom{\theta-1}{i}\right] \tag{12}$$

$$E(X^3)=\sum_{i=0}^{\infty}\sum_{j=0}^{i}\binom{i}{j}(-1)^i\frac{(j+4)!}{(i+1)^{(j+5)}}\left[\frac{(n-k)\alpha}{n}\binom{\alpha-1}{i}+\frac{k\theta}{n}\binom{\theta-1}{i}\right] \tag{13}$$

Method of Maximum Likelihood:

One sees from (6) that moment estimates for the parameters of the EG distribution with presence of *k* outliers can not be obtained in closed forms and therefore that is little point in considering the method any further. Proceeding with the method of maximum likelihood, the log likelihood function from a sample of *n* observations, $(X_1, X_2, ..., X_n)$ is given by

$$\begin{aligned} L(\alpha,\theta) &= \ln f(x_1,x_2,...,x_n;\alpha,\theta) \\ &= (n-k)\ln\alpha + k\ln\theta - \ln c(n,k) + \sum_{i=1}^{n}\ln x_i - \sum_{i=1}^{n} x_i \\ &+ (\alpha-1)\sum_{i=1}^{n}\ln\left(1-e^{-x_i}(1+x_i)\right) + \ln\sum_{\underline{A}}\prod_{r=1}^{k}\left(1-e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha} \end{aligned} \tag{14}$$

where $c(n,k)=n!/((n-k)!k!)$.

Taking the derivative with respect to α and θ and equating to 0, we obtain the normal equations as

$$\frac{\partial L(\alpha,\theta)}{\partial\alpha}=\frac{n-k}{\alpha}+\sum_{i=1}^{n}\ln\left(1-e^{-x_i}(1+x_i)\right)-\frac{\frac{\partial}{\partial\alpha}\sum_{\underline{A}}\prod_{r=1}^{k}\left(1-e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha}}{\sum_{\underline{A}}\prod_{r=1}^{k}\left(1-e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha}}$$

$$=\frac{n-k}{\alpha}+\sum_{i=1}^{n}\ln\left(1-e^{-x_i}(1+x_i)\right)-\frac{\sum_{\underline{A}}\prod_{r=1}^{k}\left[\left(1-e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha}\ln\left(1-e^{-x_{A_r}}(1+x_{A_r})\right)\right]}{\sum_{\underline{A}}\prod_{r=1}^{k}\left(1-e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha}}\overset{set}{=}0 \tag{15}$$

$$\frac{\partial L(\alpha,\theta)}{\partial\theta}=\frac{k}{\theta}+\frac{\frac{\partial}{\partial\theta}\sum_{\underline{A}}\prod_{r=1}^{k}\left(1-e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha}}{\sum_{\underline{A}}\prod_{r=1}^{k}\left(1-e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha}}$$

$$=\frac{k}{\theta}+\frac{\sum_{\underline{A}}\prod_{r=1}^{k}\left[\left(1-e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha}\ln\left(1-e^{-x_{A_r}}(1+x_{A_r})\right)\right]}{\sum_{\underline{A}}\prod_{r=1}^{k}\left(1-e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha}}\overset{set}{=}0 \tag{16}$$

Here, we need to use either the scoring algorithm or the Newton-Raphson algorithm to solve the two non-linear equations simultaneously, so we will solve for $\hat{\alpha}$ and $\hat{\theta}$ iteratively, using the Newton- Raphson method, a tangent method for root finding. In our case we will estimate $\beta=(\alpha,\theta)$ iteratively:

$$\hat{\beta}_{i+1}=\hat{\beta}_i-G^{-1}g\ , \tag{17}$$

where g is the vector of normal equations for which we want

$$g = [g_1 \quad g_2] \ , \tag{18}$$

with

$$g_1 = \frac{n-k}{\alpha} + \sum_{i=1}^{n} \ln\left(1 - e^{-x_i}(1+x_i)\right) - \frac{\sum_{\underline{A}} \prod_{r=1}^{k} \left[\left(1 - e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha} \ln\left(1 - e^{-x_{A_r}}(1+x_{A_r})\right) \right]}{\sum_{\underline{A}} \prod_{r=1}^{k} \left(1 - e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha}} , \tag{19}$$

$$g_2 = \frac{k}{\theta} + \frac{\sum_{\underline{A}} \prod_{r=1}^{k} \left[\left(1 - e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha} \ln\left(1 - e^{-x_{A_r}}(1+x_{A_r})\right) \right]}{\sum_{\underline{A}} \prod_{r=1}^{k} \left(1 - e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha}} , \tag{20}$$

and G is the matrix of second derivatives

$$G = \begin{bmatrix} \frac{d\,g_1}{d\alpha} & \frac{d\,g_1}{d\theta} \\ \frac{dg_2}{d\alpha} & \frac{dg_2}{d\theta} \end{bmatrix} \tag{21}$$

where

$$\frac{\partial g_1}{\partial \alpha} = \frac{k-n}{\alpha^2} + \frac{\sum_{\underline{A}} \prod_{r=1}^{k} \left[\left(1 - e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha} \cdot \left(\ln\left(1 - e^{-x_{A_r}}(1+x_{A_r})\right)\right)^2 \right]}{\sum_{\underline{A}} \prod_{r=1}^{k} \left(1 - e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha}}$$
$$- \left(\frac{\sum_{\underline{A}} \prod_{r=1}^{k} \left[\left(1 - e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha} \cdot \ln\left(1 - e^{-x_{A_r}}(1+x_{A_r})\right) \right]}{\sum_{\underline{A}} \prod_{r=1}^{k} \left(1 - e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha}} \right)^2 , \tag{22}$$

$$\frac{\partial g_1}{\partial \theta} = \frac{\partial g_2}{\partial \alpha} = - \frac{\sum_{\underline{A}} \prod_{r=1}^{k} \left[\left(1 - e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha} \cdot \left(\ln\left(1 - e^{-x_{A_r}}(1+x_{A_r})\right)\right)^2 \right]}{\sum_{\underline{A}} \prod_{r=1}^{k} \left(1 - e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha}}$$
$$+ \left(\frac{\sum_{\underline{A}} \prod_{r=1}^{k} \left[\left(1 - e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha} \cdot \ln\left(1 - e^{-x_{A_r}}(1+x_{A_r})\right) \right]}{\sum_{\underline{A}} \prod_{r=1}^{k} \left(1 - e^{-x_{A_r}}(1+x_{A_r})\right)^{\theta-\alpha}} \right)^2 , \tag{23}$$

$$\frac{\partial g_2}{\partial \theta} = \frac{-1}{\theta^2} + \frac{\sum_{\underline{A}} \prod_{r=1}^{k} \left[\left(1 - e^{-x_{A_r}} (1 + x_{A_r}) \right)^{\theta - \alpha} \cdot \left(\ln \left(1 - e^{-x_{A_r}} (1 + x_{A_r}) \right) \right)^2 \right]}{\sum_{\underline{A}} \prod_{r=1}^{k} \left(1 - e^{-x_{A_r}} (1 + x_{A_r}) \right)^{\theta - \alpha}}$$

$$- \left(\frac{\sum_{\underline{A}} \prod_{r=1}^{k} \left[\left(1 - e^{-x_{A_r}} (1 + x_{A_r}) \right)^{\theta - \alpha} \cdot \ln \left(1 - e^{-x_{A_r}} (1 + x_{A_r}) \right) \right]}{\sum_{\underline{A}} \prod_{r=1}^{k} \left(1 - e^{-x_{A_r}} (1 + x_{A_r}) \right)^{\theta - \alpha}} \right)^2 \tag{24}$$

The Newton-Raphson algorithm converges, as our estimates of α and θ change by less than a tolerated amount with each successive iteration, to $\hat{\alpha}$ and $\hat{\theta}$.

Note that for $\alpha = \theta = \beta$, in case of no outlier presence (k=0), $\hat{\beta}$ can be obtain as

$$\hat{\beta} = \frac{-n}{\sum_{i=1}^{n} \ln \left(1 - e^{-x_i} (1 + x_i) \right)}$$

it is given by Gupta et al. (1998), also for k=1 it is obtained by Shadrokh and Pazira (2010).

Numerical Experiments and Discussions:

In this paper, we have addressed the problem of estimating parameters of Exponentiated Gamma distribution in presence of k outliers. In order to have some idea about Bias and Mean Square Error (MSE) of methods of moment and MLE, we perform sampling experiments using a MATLAB. The results are given in Tables 1, 2, 3 and 4, and Figures 2, 3, 4 and 5, for α=2 and θ=4. We report the average estimates and the MSEs based on *1000* replications. It is observed that the maximum likelihood estimator work quit well.

Table 1: $\alpha = 2$, $\theta = 4$ *and* $k = 1$

n	Bias $\hat{\alpha}_{MOM}$	MSE $\hat{\alpha}_{MOM}$	Bias $\hat{\alpha}_{MLE}$	MSE $\hat{\alpha}_{MLE}$
10	- 0.0799	0.0060	0.0621	0.0051
15	-0.0422	0.0060	0.0196	0.0051
20	-0.0286	0.0041	0.0438	0.0037
25	-0.0226	0.0029	0.0156	0.0027
30	-0.0167	0.0021	0.0105	0.0012
35	-0.0161	0.0018	0.0110	0.0017
40	-0.0137	0.0016	0.0098	0.0016
45	-0.0099	0.0014	0.0059	0.0013
50	-0.0035	0.0010	0.0036	0.0009

Table 2: $\alpha = 2$, $\theta = 4$ *and* $k = 1$

n	Bias $\hat{\theta}_{MOM}$	MSE $\hat{\theta}_{MOM}$	Bias $\hat{\theta}_{MLE}$	MSE $\hat{\theta}_{MLE}$
10	-0.0807	0.0092	0.0421	0.0625
15	-0.1131	0.0107	0.0037	0.0038
20	-0.0859	0.0054	0.0144	0.0030
25	-0.0039	0.0029	0.0025	0.0027
30	-0.0288	0.0027	0.0129	0.0021
35	-0.0254	0.0023	0.0091	0.0018
40	-0.0222	0.0019	0.0074	0.0016
45	-0.0204	0.0016	0.0060	0.0013
50	-0.0186	0.0013	0.0047	0.0008

Table 3: $\alpha = 2$, $\theta = 4$ *and* $k = 2$

n	Bias $\hat{\alpha}_{MOM}$	MSE $\hat{\alpha}_{MOM}$	Bias $\hat{\alpha}_{MLE}$	MSE $\hat{\alpha}_{MLE}$
10	-0.857	0.636	0.788	0.562
15	-0.928	0.408	0.668	0.329
20	-0.845	0.257	0.569	0.203
25	-0.565	0.164	0.514	0.127
30	-0.325	0.128	0.493	0.098
35	-0.441	0.093	0.247	0.071
40	-0.263	0.058	0.338	0.044
45	-0.295	0.036	0.226	0.027
50	-0.258	0.023	0.183	0.017

Table 4: $\alpha = 2$, $\theta = 4$ *and* $k = 2$

n	Bias $\hat{\theta}_{MOM}$	MSE $\hat{\theta}_{MOM}$	Bias $\hat{\theta}_{MLE}$	MSE $\hat{\theta}_{MLE}$
10	-0.997	0.335	0.528	0.240
15	-0.972	0.286	0.175	0.208
20	-0.232	0.203	0.116	0.146
25	-0.188	0.157	0.751	0.113
30	-0.972	0.125	0.592	0.090
35	-0.441	0.087	0.298	0.061
40	-0.305	0.073	0.178	0.053
45	-0.192	0.056	0.168	0.041
50	-0.157	0.040	0.177	0.029

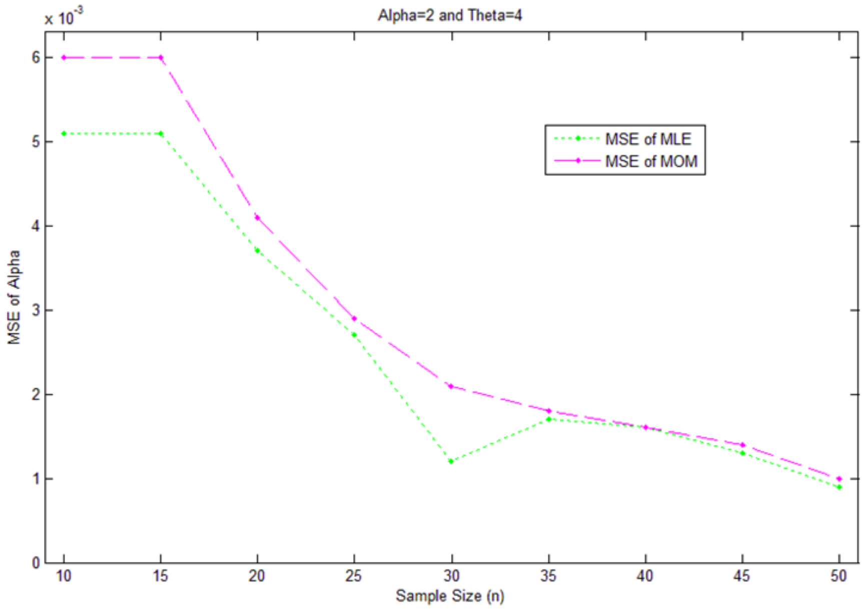

Fig. 2: MSE of the estimator of α as function of sample size for $k=1$.

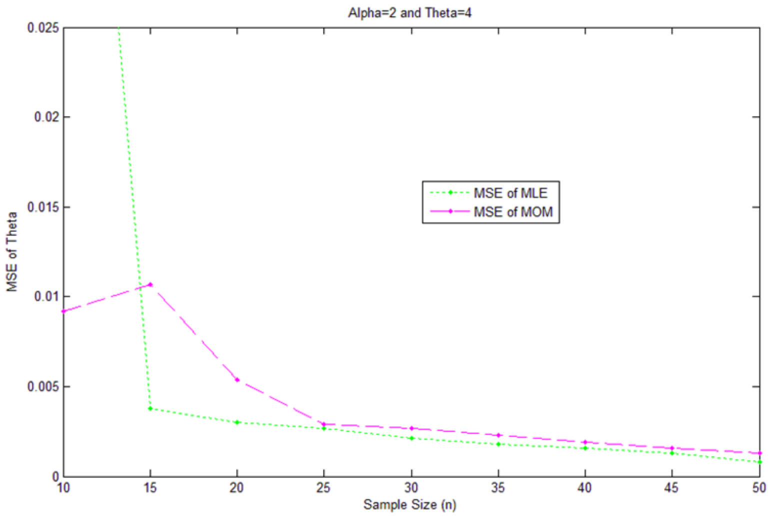

Fig. 3: MSE of the estimator of θ as function of sample size for $k=1$.

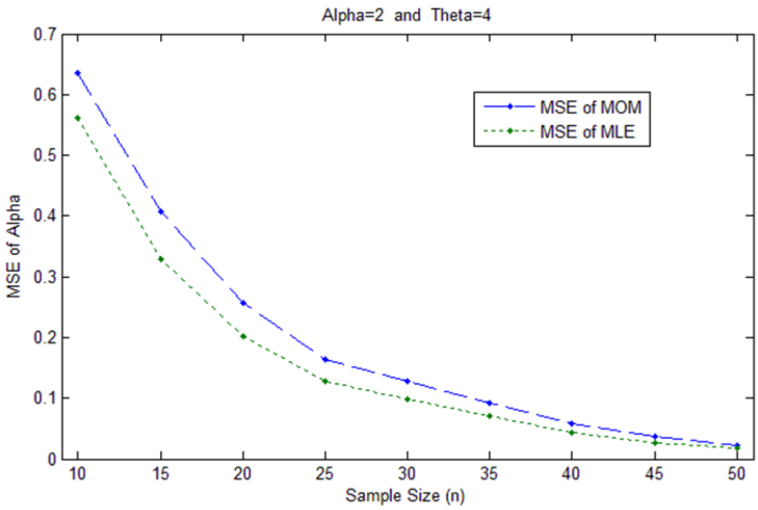

Fig. 4: MSE of the estimator of α as function of sample size for $k=2$.

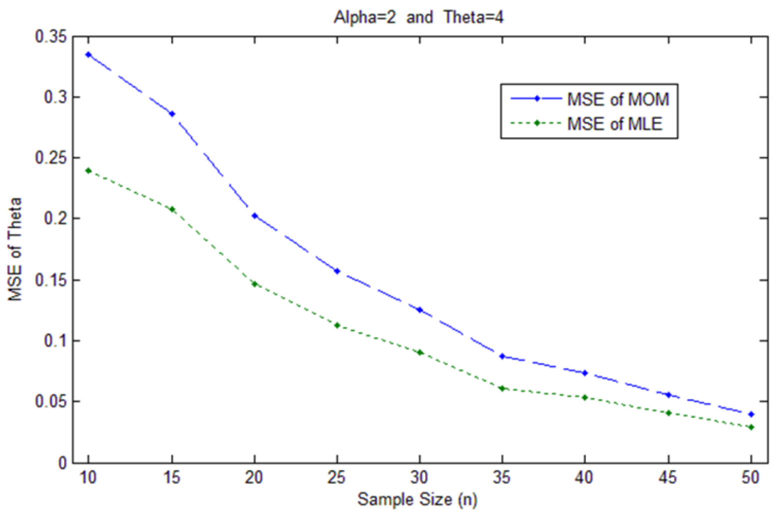

Fig. 5: MSE of the estimator of θ as function of sample size for $k=2$.

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