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An Approximation Approach to Fuzzy Numbers by Continuous Parametric Interval

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Abstract: In the present paper, the researchers discuss the problem of parametric interval approximation of fuzzy numbers. The interval which fulfills two conditions. First it is a continuous interval approximation operator. Secondly, the parametric weighted distance between this interval and the approximated number is minimal and continuous with respect to a fuzzy quantity. This interval can be used as a crisp set approximation. In this regard, a novel approach is proposed to ranking fuzzy numbers. That is, this method can effectively rank various fuzzy numbers and their images images and addresses the deficiencies of previous techniques. Some comparative examples to illustrate the advantages of the proposed method are outlined in this article.

Key words: Fuzzy number; Defuzzification; Ranking; Parametric Interval approximation.

INTRODUCTION

Approximation an interval representation of a fuzzy number may have many useful applications. By utilizing this method, it is possible to apply (in fuzzy number approaches) some results derived from interval number analysis. For example, it may be applied to a comparison of fuzzy numbers by using the order relations defined on the set of interval numbers. Various authors (Saneifard, 2009; Grzegorzewski, 2002) have studied the crisp approximation of fuzzy sets and proposed a rough theoretic definition of that crisp approximation, called the nearest interval approximation of a fuzzy set. Moreover, quite different approach to crisp approximation of fuzzy sets was applied in Chakrabarty, *et al.*, 1998, in which a rough theoretic definition of that crisp approximation, called the nearest ordinary set of a fuzzy set, was proposed and the construction of such a set suggested. Discrete fuzzy sets were discussed, however approximation of a given fuzzy set not unique. Thus this article will is not and hence, will not be discussed herein. Having reviewed the previous interval approximations, this article proposes here a method to find the parametric interval approximation of a fuzzy number, to fulfill two conditions. In the first, the interval is a continuous interval approximation operator. Secondly, the parametric distance between this interval and the approximated number is minimal and continuous. The main purpose of this article is to illustrate how a parametric interval of a fuzzy number can be utilized as a crisp set approximation of a fuzzy number. Therefore, by the employing this approximation, a new method for ranking of fuzzy numbers presented here in. In addition this method removes the ambiguities and shortcomings of previous ranking procedures addressed by others in the past. This article derive the formulae for determining the approximation interval for a fuzzy number given in a general form as well as for a fuzzy number of LR type. The paper is organized as follows: In Section 2, this article recalls some fundamental results on fuzzy numbers. In Section 3, a crisp set approximation of a fuzzy number (parametric interval approximate) is obtained. In this Section some remarks are proposed and illustrated. Proposed method for ranking fuzzy numbers is in the Section 4. Discussion and comparison of this work and other methods are carried out in Section 5. The paper ends with conclusions in Section 6.

2. Basic Definition and Notation

The basic definitions of a fuzzy number are given in (Zimmermann, 1991; Wang, *et al.,* 2001; Baldwin, 1979) as follows:

Definition 1:

Let X be a universe set. A fuzzy set A of X is defined by a membership function $\mu_A(x) \to [0,1]$, where

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$\mu_A(x)$, $\forall x \in X$ indicates the degree of x in A.

Definition 2:

A fuzzy subset A of universe set X is normal iff $sup_{x\in X}\,\mu_A(x) = 1$, where X is the universe set.

Definition 3:

A fuzzy subset A of universe set X is convex iff $\mu_A(\lambda x + (1-\lambda)y) \geq (\mu_A(x) \wedge \mu_A(y)), \forall x, y \in X, \forall \lambda \in [0,1]$.

In this article symbols $\wedge$ and $\vee$ denotes the minimum and maximum operators, respectively.

Definition 4:

A fuzzy set A is a fuzzy number iff A is normal and convex on X.

Definition 5:

For fuzzy set A Support function is defined as follows:

$supp(A) = \overline{\{x|\mu_A(x) > 0\}}$,

where $\overline{\{x|\mu_A(x) > 0\}}$ is closure of set $\{x|\mu_A(x) > 0\}$.

A space of all fuzzy numbers will be denoted by F and this article recall that

$core\,A = \{x \in \mathcal{R}|\ \mu_A(x) = 1\}$.

Definition 6:

Assume that the fuzzy number $A \in F$ is represented by means of the following representation:

$$A = \cup_{\alpha\in[0,1]}(\alpha, A_\alpha). \tag{1}$$

Here

$A_\alpha = \{x\colon\ \mu_A(x) \geq \alpha\}$,

is the a-level set of the fuzzy number A. This article consider normal and convex fuzzy numbers. Therefore the a-level sets may be represented in the form of a segment,

$$\forall \alpha \in [0,1]: A_\alpha = [L_A(\alpha), R_A(\alpha)] \subset (-\infty, +\infty), \tag{2}$$

Here, $L : [0,1] \rightarrow (-\infty, +\infty)$ is a monotonically non-decreasing left continuous function and $R : [0,1] \rightarrow (-\infty, +\infty)$ is a monotonically non-increasing right-continuous functions. The functions $L(.)$ and $R(.)$ express the left and right sides of a fuzzy number, respectively. In other words,

$$L(\alpha) = \mu_\uparrow^{-1}(\alpha)\ ,\ \ R(\alpha) = \mu_\downarrow^{-1}(\alpha), \tag{3}$$

where $L(\alpha) = \mu_\uparrow^{-1}(\alpha)$ and $R(\alpha) = \mu_\downarrow^{-1}(\alpha)$, denote quasi-inverse functions of the increasing and decreasing parts of the membership functions $\mu(t)$, respectively. As a result, the decomposition representation of the fuzzy number A, called the L-R representation, has the following form:

$$A = \bigcup_{\alpha\in[0,1]} (\alpha, [L_A(\alpha), R_A(\alpha)]).$$

Definition 7:

(Nasibove, 2007).

The following values constitute the weighted averaged representative and weighted width, respectively, of the fuzzy number:

$$A = \bigcup_{\alpha\in[0,1]} (\alpha, [L_A(\alpha), R_A(\alpha)]). \tag{4}$$

and

$$D(A) = \int_0^1 (R_A(\alpha) - L_A(\alpha))p(\alpha)d\alpha \tag{5}$$

Here $0 \le c \le 1$ denotes an "optimism/pessimism" coefficient in conducting operations on fuzzy numbers. The function $p: [0,1] \to [0, +\infty)$ denotes the distribution density of the importance of the degrees of fuzziness, where $\int_0^1 p(\alpha)d\alpha = 1$. In particular cases, it may be assumed that

$p(\alpha) = (k+1)\alpha^k, \quad k = 0,1,2,\cdots$

Definition 8:
(Nasibove, 2007).

For arbitrary fuzzy numbers *A* and *B* the quantity

$$d_p(A,B) = \sqrt{[I(A) - I(B)]^2 + [D(A) - D(B)]^2}, \tag{6}$$

is called the parametric distance between the fuzzy numbers *A* and *B*.

Definition 9:
(Grzegorzewski, 2002).

An operator $I: F \to (\text{Set of closed intervals in } \mathcal{R})$ is called an interval approximation operator if for any A $\in F$

(á) $I(A) \subseteq supp(A)$,

(ḃ) $core(A) \subseteq I(A)$,

(ć) $\forall(\varepsilon > 0)\, \exists(\delta > 0)\; S.t\; d(u,v) < \delta \Rightarrow d(I(u), I(v)) < \varepsilon$,

where $d: F \to [0, +\infty[$ denotes a metric defined in the family of all fuzzy numbers.

Definition 10:
(Grzegorzewski, 2002).

An interval approximation operator satisfying in condition (ć) for any *A*,*B*$\in$*F* is called the continuous interval approximation operator.

Nearest Weighted Interval of a Fuzzy Number:

Various authors in (Chakrabarty,1998; Grzegorzewski, 2002) have studied the crisp approximation of fuzzy sets. They proposed a rough theoretic definition of that crisp approximation, called the nearest ordinary set and nearest interval approximation of a fuzzy set. In this Section, the researchers will propose another approximation called the parametric interval approximation.

Let *A* be an arbitrary fuzzy number and $[L_A(a), R_A(a)]$ be its *a*-cut set. This effort attempts to find a closed interval $C_{d_p}(A)$, which is the parametric interval to A with respect to metric d_p. Since each interval with constant α-cuts for all $\alpha\in(0,1]$ is a fuzzy number, hence, suppose $C_{d_p}(A)=[L_C,R_C]$, i.e. $(C_{d_p}(A))_\alpha=[L_C,R_C],\ \forall\alpha\in(0,1]$. So, this article has to minimize

$$d_p\left(A,C_{d_p}(A)\right)=\left(\left[I(A)-I(C_{d_p}(A))\right]^2+\left[D(A)-D(C_{d_p}(A))\right]^2\right)^{\frac{1}{2}} \tag{7}$$

with respect to L_C and R_C, where

$I\left(C_{d_p}(A)\right)=\int_0^1(cL_C+(1-c)R_C)p(\alpha)d\alpha$, $D\left(C_{d_p}(A)\right)=\int_0^1(R_C-L_C)p(\alpha)d\alpha$

In order to minimize d_p it suffices to minimize

$\overline{D}_p(L_C,R_C)=d_p^2(L_C,R_C)$.

It is clear that, the parameters L_C and R_C which minimize Eq.(7) must satisfy in

$\nabla\overline{D}_p(L_C,R_C)=\left(\frac{\partial\overline{D}_p}{\partial L_C},\frac{\partial\overline{D}_p}{\partial R_C}\right)=0.$

Therefore, the following equations are utilized in this endeavor:

$$\begin{cases}\dfrac{\partial\overline{D}_p(L_C,R_C)}{\partial L_C}=-2c\displaystyle\int_0^1(c(L_A(\alpha)-L_C)+(1-c)(R_A(\alpha)-R_C))p(\alpha)d\alpha\\ \qquad\qquad +2\displaystyle\int_0^1((R_A(\alpha)-R_C)-(L_A(\alpha)-L_C))p(\alpha)d\alpha=0,\\ \dfrac{\partial\overline{D}_p(L_C,R_C)}{\partial L_C}=-2(1-c)\displaystyle\int_0^1(c(L_A(\alpha)-L_C)+(1-c)(R_A(\alpha)-R_C))p(\alpha)d\alpha\\ \qquad\qquad -2\displaystyle\int_0^1((R_A(\alpha)-R_C)-(L_A(\alpha)-L_C))p(\alpha)d\alpha=0.\end{cases} \tag{8}$$

The parameters L_C associated with the left bound and R_C associated with the right bound of the parametric interval can be found by using Eq. (8) as follows:

$$\begin{cases}L_C=\displaystyle\int_0^1 L_A(\alpha)p(\alpha)d\alpha,\\ R_C=\displaystyle\int_0^1 R_A(\alpha)p(\alpha)d\alpha.\end{cases} \tag{9}$$

Remark 1:

Since, $det\begin{bmatrix}\frac{\partial\overline{D}_p^2(L_C,R_C)}{\partial L_C^2} & \frac{\partial\overline{D}_p^2(L_C,R_C)}{\partial R_C\partial L_C}\\ \frac{\partial\overline{D}_p^2(L_C,R_C)}{\partial L_C\partial R_C} & \frac{\partial\overline{D}_p^2(L_C,R_C)}{\partial R_C^2}\end{bmatrix}=det\begin{bmatrix}2c^2 & 2c(1-c)-2\\ 2c(1-c)-2 & 2(1-c)^2+2\end{bmatrix}=4>0$ and $\forall c\in[0,1]$, $\frac{\partial\overline{D}_p^2(L_C,R_C)}{\partial L_C^2}=1>0$, therefore L_C and R_C given by (9), minimize $d_p\left(A,CI_{d_p}(A)\right)$. Therefore, the interval

$$C_{d_p}(A)=\left[\int_0^1 L_A(\alpha)p(\alpha)d\alpha,\int_0^1 R_A(\alpha)p(\alpha)d\alpha\right] \tag{10}$$

is the nearest parametric interval approximation of fuzzy number A with respect to metric d_p.

Remark 2:

Whenever, in the distribution density function $p(\alpha) = (k+1)\alpha^k$, where k=1, therefore $C_{d_p}(A)$ is weighted interval-value possibilistic mean (Carlsson, *et al.*, 2001).

Remark 3:

If, in distribution density function $p(a)=(k+1)a^k$, assuming k=0, therefore $C_{d_p}(A)$ is the expected interval (Grzegorzewski, 2002).

However the intention of, this article is to approximate a fuzzy number using a crisp interval. Thus, the researchers have used an operator $C_{d_p}(A): F \to (\textit{set of closed intervals in } \mathcal{R})$ which transforms fuzzy numbers into a family of closed intervals on the real line.

Theorem 1:

The operator $C_{d_p}(A): F \to (\textit{set of closed intervals in } \mathcal{R})$ is an interval approximation operator, i.e. $C_{d_p}(A)$ is a continuous interval approximation operator.

Proof:

It is easy to verify that the conditions (a') and (b') are valid. For the Proof of (c'), let A and B represent two fuzzy numbers with parametric intervals $C_{d_p}(A) = [L_C(A), R_C(A)]$ and $C_{d_p}(B) = [L_C(B), R_C(B)]$, respectively. Then

$$d_p^2\left(C_{d_p}(A), C_{d_p}(B)\right) = \left[I\left(C_{d_p}(A)\right) - I\left(C_{d_p}(B)\right)\right]^2 + \left[D\left(C_{d_p}(A)\right) - D\left(C_{d_p}(B)\right)\right]^2$$

$$= \left[\int_0^1 (cL_C(A) + (1-c)R_C(A))p(\alpha)d\alpha - \int_0^1 (cL_C(B) + (1-c)R_C(B))p(\alpha)d\alpha\right]^2$$

$$+ \left[\int_0^1 (R_C(A) + L_C(A))p(\alpha)d\alpha - \int_0^1 (R_C(B) + L_C(B))p(\alpha)d\alpha\right]^2$$

$$= \left[c\big(L_C(A) - L_C(B)\big) + (1-c)\big(R_C(A) - R_C(B)\big)\right]^2 + \left[\big(R_C(A) - R_C(B)\big) - \big(L_C(A) - L_C(B)\big)\right]^2$$

$$= \left[c\left(\int_0^1 L_A(\alpha)p(\alpha)d\alpha - \int_0^1 L_B(\alpha)p(\alpha)d\alpha\right) + (1 - c)\left(\int_0^1 R_A(\alpha)p(\alpha)d\alpha - \int_0^1 R_B(\alpha)p(\alpha)d\alpha\right)\right]^2$$

$$+ \left[\left(\int_0^1 R_A(\alpha)p(\alpha)d\alpha - \int_0^1 R_B(\alpha)p(\alpha)d\alpha\right) + \left(\int_0^1 L_A(\alpha)p(\alpha)d\alpha - \int_0^1 L_B(\alpha)p(\alpha)d\alpha\right)\right]^2$$

$$= \left[c\int_0^1 (L_A(\alpha) - L_B(\alpha))p(\alpha)d\alpha + (1-c)\int_0^1 (R_A(\alpha) - R_B(\alpha))p(\alpha)d\alpha\right]^2$$

$$+ \left[\int_0^1 (R_A(\alpha) - R_B(\alpha))p(\alpha)d\alpha - \int_0^1 (L_A(\alpha) - L_B(\alpha))p(\alpha)d\alpha\right]^2 = d_p^2(A,B).$$

It means that $\forall \varepsilon > 0, \exists \delta = \varepsilon > 0$, when $d_p(A,B) < \delta$, then, this article has $d_p\left(C_{d_p}(A), C_{d_p}(B)\right) < \varepsilon$ showing that the parametric interval approximation is continuous interval approximation.

In other words, if *A* and *B* are close enough, then their parametric interval approximations obtained by C_{d_p} are also close enough.

The Preference Ordering of Fuzzy Numbers:

In this Section, the researchers propose a novel technique for ranking of fuzzy numbers associated with the parametric interval.

If *A* is an arbitrary fuzzy number and $[L_A(\alpha), R_A(\alpha)]$ be its *α*-cut then C_{d_p} be its the parametric interval.

Since every parametric interval can be used as a crisp set approximation of a fuzzy number, therefore, the resulting interval is used to rank the fuzzy numbers. Thus, C_{d_p} is used to rank fuzzy numbers.

Definition 11:

Let $\mathcal{F}[L]$ represents the set of closed intervals in *R*. The interval order is as follows:

$\forall a_1, b_1, a_2, b_2 \in \mathcal{R}$ verifying $a_1 \leq b_1$ & $a_2 \leq b_2$

$[a_1, b_1] \leq [a_2, b_2] \Leftrightarrow a_1 \leq a_2$ & $b_1 \leq b_2$,

Let *A* and $\mathcal{B} \in \mathcal{F}[L]$, this article also defines $\mathcal{MAX}(\mathcal{A},\mathcal{B})$ and $\mathcal{MIN}(\mathcal{A},\mathcal{B})$ the maximum and the minimum of two intervals as follows:

$\mathcal{MAX}(\mathcal{A},\mathcal{B}) = \mathcal{A} \vee \mathcal{B}$,

$\mathcal{MIN}(\mathcal{A},\mathcal{B}) = \mathcal{A} \wedge \mathcal{B}$,

where the maximum and the minimum of the intervals are defined in the following way:

$\forall a_1, b_1, a_2, b_2 \in \mathcal{R}$ verifying $a_1 \leq b_1$ & $a_2 \leq b_2$,

$[a_1, b_1] \vee [a_2, b_2] = [a_1 \vee a_2, b_1 \vee b_2]$,

$[a_1, b_1] \wedge [a_2, b_2] = [a_1 \wedge a_2, b_1 \wedge b_2]$.

Let *A* and *B*∈*F* denote two arbitrary fuzzy numbers, and $C_{d_p}(A) = [a_1, a_2]$ and $C_{d_p}(B) = [b_1, b_2]$ express the parametric intervals of *A* and *B*, respectively. Define the ranking of *A* and *B* by $C_{d_p}(.)$ on *F*, are defined as follows

(1) $\mathcal{MIN}(\mathcal{A},\mathcal{B}) = \mathcal{MAX}(\mathcal{A},\mathcal{B})$ if only if $A \sim B$,

(2) $\mathcal{MIN}(\mathcal{A},\mathcal{B}) < MAX(\mathcal{A},\mathcal{B})$ if only if *A*<*B* ,

(3) $C_{d_p}(A) \supset C_{d_p}(B)$ if only if *A*>*B*.

Then, this attempt formulates the order $\gtrsim$ and $\lesssim$ as $A \gtrsim B$ if and only if *A*>*B* or *A*~*B*, and $A \lesssim B$ if and only if *A*<*B* or *A*~*B*.

Remark 4:

For two arbitrary fuzzy numbers *A* and *B*, this paper states that

$C_{d_p}(A+B) = C_{d_p}(A) + C_{d_p}(B)$

Proof:

Let $[L_A(\alpha), R_A(\alpha)]$ and $[L_B(\alpha), R_B(\alpha)]$ be the *α*-cut sets of *A* and *B*, respectively. Therefore, $C_{d_p}(A) + C_{d_p}(B)$

$$= \left[\int_0^1 L_A(\alpha)p(\alpha)d\alpha, \int_0^1 R_A(\alpha)p(\alpha)d\alpha\right] + \left[\int_0^1 L_B(\alpha)p(\alpha)\, d\alpha, \int_0^1 R_B(\alpha)p(\alpha)d\alpha\right]$$

$$= \left[\int_0^1 (L_A(\alpha) + L_B(\alpha))p(\alpha)d\alpha, \int_0^1 (R_A(\alpha) + R_B(\alpha))p(\alpha)d\,\alpha\right]$$

$$= \left[\int_0^1 (L_A + L_B)(\alpha)p(\alpha)d\alpha, \int_0^1 (R_A + R_B)(\alpha)p(\alpha)d\alpha\right]$$

$$= C_{d_p}(A + B) \; .$$

This article considers the following reasonable axioms that Wang and Kerre [8] proposed for ranking fuzzy numbers

Let $C_{d_p}(.)$ be an ordering method, *S* the set of fuzzy quantities for which the method $C_{d_p}(.)$ can be applied and *A* a finite subset of *S*. The statement "two elements *A* and *B* in *A* satisfy that *A* has a higher ranking than *B* when $C_{d_p}(.)$ is applied to the fuzzy quantities in *A*" will be written as " *A*>*B* by $C_{d_p}(.)$ on *A*", "*A*~*B* by $C_{d_p}(.)$ on *A* ", and " $A \gtrsim B$ $C_{d_p}(.)$ on *A* " are similarly interpreted. The axioms as the reasonable properties of ordering fuzzy quantities for an ordering approach $C_{d_p}(.)$ are as follows:

A-1 For an arbitrary finite subset *A* of *S* and *A*∈*A* ; $A \gtrsim A$.

A-2 For an arbitrary finite subset *A* of *S* and $(A,B) \in \mathcal{A}^2$; $A \gtrsim B$ and $B \gtrsim A$, we should have *A*~*B*.

A-3 For an arbitrary finite subset *A* of *S* and $(A,B,C) \in \mathcal{A}^3$; $A \gtrsim B$ and $B \gtrsim C$, we should have $A \gtrsim C$.

A-4 For an arbitrary finite subset *A* of *S* and $(A,B) \in \mathcal{A}^2$; $\inf supp\,(A) > \sup supp\,(B)$, we should have $A \gtrsim B$.

A'-4 For an arbitrary finite subset *A* of *S* and $(A,B) \in \mathcal{A}^2$; $\inf supp\,(A) > \sup supp\,(B)$, we should have *A*>*B*.

A-5 Let *S* and S' be two arbitrary finite sets of fuzzy quantities in which $C_{d_p}(.)$ can be applied and *A* and *B* are in $S \cap S'$. We obtain the ranking order $A \gtrsim B$ by $C_{d_p}(.)$ on S' iff $A \gtrsim B$ by $C_{d_p}(.)$ on *S*.

A-6 Let *A*, *B*, *A*+*C* and *B*+*C* be elements of S. If $A \gtrsim B$, then $A + C \gtrsim B + C$ by $C_{d_p}(.)$ on A + C and B + C.

A'-6 Let *A*,*B*, *A*+*C* and *B*+*C* be elements of *S*. If *A*>*B* by $C_{d_p}(.)$ on *A* and *B*, then *A*+*C*>*B*+*C* by $C_{d_p}(.)$ on *A*+*C* and *B*+*C*.

A-7 For an arbitrary finite subset *A* of *S* and *A*∈*A*, the must belong to its support. $C_{d_p}(.)$.

Theorem 2:

The function $C_{d_p}(.)$ has the properties (*A*-1), (*A*-2),...,(*A*-5).

Proof:

It is easy to verify that the properties (*A*-1), (*A*-2),...,(*A*-5) and (*A*-7) are valid. To prove (*A*-6), this article considers the fuzzy numbers *A*,*B* and *C*.

Let $A \gtrsim B$, from the relation (10), there is

$C_{d_p}(A) \geq C_{d_p}(B)$,

by adding $C_{d_p}(C)$,

$C_{d_p}(A) + C_{d_p}(C) \geq C_{d_p}(B) + C_{d_p}(C)$,

and by Remark (4),

$C_{d_p}(A + C) \geq C_{d_p}(B + C)$,

Therefore

$A + C \geq B + C$.

which completes the proof. Similarly $(A' - 6)$ is hold.

Remark 5:

If $A \precsim B$, then $-A \succsim -B$.

Hence, this article can infer ranking order of the images of the fuzzy numbers.

5. Numerical Examples:

In this Section, this study compares the proposed method with others in [Abbasbandy, 2009; Saneifard, 2009; Chu, *et al.*, 2002; Chen, 1985; Wang, *et al.*, 2005]. Throughout this Section the researchers assumed that k=1 i.e. $p(a)=2a$.

Example 1:

Consider the data used in (Abbasbandy, 2006), i.e. the three fuzzy numbers, A=(6,1,1), B=(6,0,1,1), C=(6,0,1), as shown in Fig. 1.

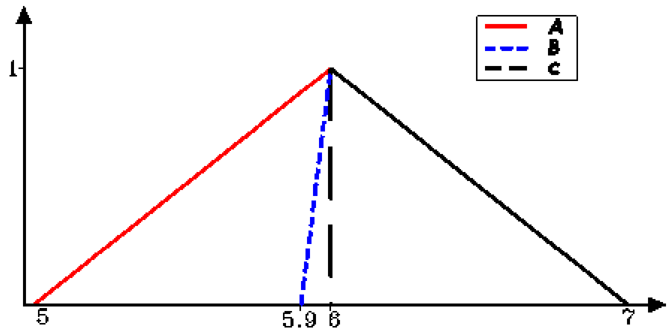

Fig. 1:

According to Eq. (10), the ranking index values are obtained, i.e., $C_{d_p}(A) = [2.83,3.16]$, $C_{d_p}(B) = [2.98,3.16]$ and $C_{d_p}(C) = [3,3.16]$. Thus, the ranking order of fuzzy numbers is C>B>A. However, by Chu and Tsao's Approach (Chu, *et al.*, 2002), the ranking order is B>C>A. Meanwhile, using the proposed CV index proposed, the ranking order is A>B>C. From Figure (1), it is obvious that the ranking results obtained by the existing approaches (Cheng, 1999; Chu, *et al.*, 2002) are unreasonable and inconsistent. On the other hand, in (Abbasbandy, 2006), the ranking result is C>B>A, which is the same as the one obtained by the proposed method. However the authors approach proves to be simpler in the computation procedure. Based on the analysis results from (Abbasbandy, 2006), the ranking results of this effort and other approaches are listed in

Table 1.
Table 1: Comparative results of Example (1)

Fuzzy number	New approach	Sign Distance with p=1	Sign Distance with p=2	Chu-Tsao	Cheng Distance	*CV* index
A	[2.83,3.16]	6.12	8.52	3.000	6.021	0.028
B	[2.98,3.16]	12.45	8.82	3.126	6.349	0.009
C	[3.00,3.16	12.50	8.85	3.085	6.351	0.008
Results	*C>B>A*	*C>B>A*	*C>B>A*	*B>C>A*	*C>B>A*	*B>C>A*

Example 2:

Observe the two symmetric triangular fuzzy numbers A=_(0.5,0.3,0.3) and B=(0.5,0.1,0.1), as shown in Fig. 2.

Through the proposed approach in this article, the ranking index values can be obtained as $C_{d_p}(A) = [0.2,0.3]$, and $C_{d_p}(B) = [0.23,0.26]$. Then, the ranking order of fuzzy numbers result in *A>B.*

As fuzzy numbers *A* and *B* have the same mode and symmetric spread, most of the existing methods fail to produce proper results. For instance, in (Abbasbandy, 2006), different ranking orders are obtained when different index values (*p*) are used. When *p*=1 and *p*=2, the ranking order of fuzzy numbers is *A~B* and *A>B*, respectively. Meanwhile, using the approaches in (Chu, *et al.*, 2002; Chen, 1985), the ranking order is the same, i.e., *A~B*. Nevertheless, inconsistent results were produced when the distance index and CV index of Cheng's approach are respectively employed. Moreover, the ranking order obtain by Wang's approach (Wang, *et al.*, 2005) is *A>B*. Tran, *et al.*, 2002 attains conflict results when D_{max} and D_{min} are respectively used. Additionally, by the approaches provided in (Liu, 2001; Matarazzo, *et al.*, 2001), different orders are achieved order is obtained when various indices of optimal values are used. However, decision makers prefer to the result *A>B* intuitionally

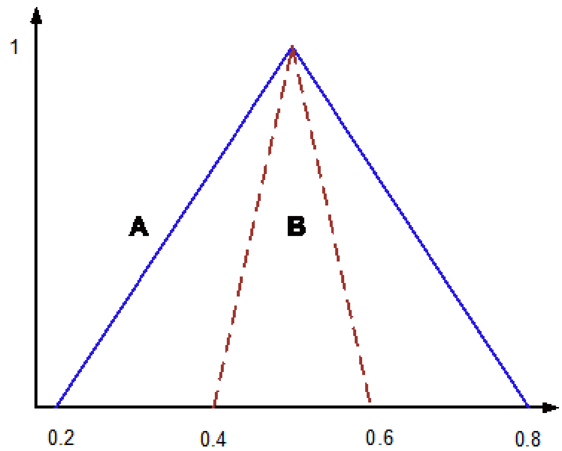

Fig. 2:

Example 3:

Consider the three fuzzy numbers A=(1,2,5), B=(0,3,4) and C=(2,2.5,3), (see Fig. 3).

By using this new approach $C_{d_p}(A) = [0.83,1.5]$, $C_{d_p}(B) = [1,1.66]$ and $C_{d_p}(C) = [1.16,1.33]$. Hence, the ranking order is *C<B<A* too. To compare this outcome with other methods in (Chu, *et al.*, 2002), the reader is detected to Table 2.

Furthermore, to aforesaid example $C_{d_p}(-A) = [-1.5,-0.83]$, $C_{d_p}(-B) = [-1.66,-1]$ and $C_{d_p}(-C) = [-1.33,-1.16]$, consequently the ranking order of the images of three fuzzy number is -*B*<-*A*<-*C*. Clearly, this proposed method has consistency in ranking fuzzy numbers and their images, which could not be guaranteed by CV-index method.

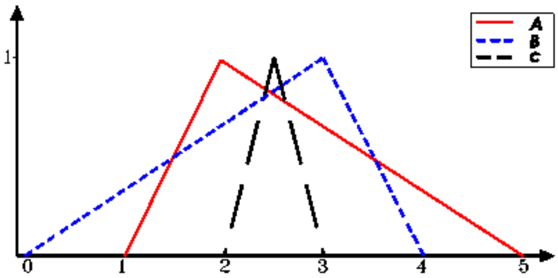

Fig. 3:

Table 2: Comparative results of Example (3)

Fuzzy number	New approach	Sign Distance with p=1	Sign Distance with p=2	Distance Minimization	Chu and Tsao
A	[0.83,1.50]	3	2.16	2.50	0.74
B	[1.00,1.66]	3	2.70	2.50	0.74
C	[1.16,1.33]	3	2.70	2.50	0.75
Results	*C*<*A*<*B*	*C*~*A*~*B*	*C*<*A*~*B*	*C*~*A*~*B*	*A*~*B*<

All the above examples show that this method is more consistent with institution than the previous ranking methods. This method can overcome the shortcomings of other methods.

Conclusion:

In this study, the researchers discuss the problem of parametric interval approximation of fuzzy numbers. This interval can be used as a crisp set approximation with respect to a fuzzy quantity. Then, by using this, the researchers proposed a novel approach to ranking fuzzy numbers. This method can effectively rank various fuzzy numbers, their images and overcome the shortcomings of the previous techniques.

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