Design of Adaptive Robust Controller based on Type-2 Fuzzy Logic Systems

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Abstract: This paper presents a fuzzy robust controller for the nonlinear systems with uncertain dynamics. The main idea of this method is that Type-2 fuzzy system estimates nonlinear and uncertain functions exist in the system equations and then the parameters of type-2 fuzzy system are tuned online by adaptive rules obtained from Lyapunov theory and finally, stability and convergence analysis will be carried out to determine the tracking performance. At the end, simulations are implemented for an individual inverted pendulum model in two cases: certain and uncertain equations of the system. In both cases, the simulation results show that proposed tracking controller has a better performance in tracking and robustness than type-1 fuzzy controller.

Key words: Adaptive-fuzzy control; Interval Type-2 fuzzy logic systems; Robust control; Tracking control; Uncertain rule base;

INTRODUCTION

Nonlinear systems control has been important issue from many years. Conventionally, control system design is obtained from mathematical models based on physical laws. But, in fact, most of the parameters and structures in a system are unknown due to ambient changes, modeling errors and dynamical parameters which cannot be modeled. To solve these problems in design of control systems, some methods and techniques based on intelligence technology have been introduced in recent years. Individually, fuzzy systems have been developed successfully to control complicated processes whose mathematical models are obtained hardly. An overview about a system is that it can have either unknown or uncertain equations. Actually, in addition to unknown mathematical model of the system, they are uncertain too. Many efforts have been made to control these systems, e.g., a combination of neural networks and fuzzy logic methods (like if-then statements) or fuzzy logic and evolutionary algorithms. It is obvious that our world is based on probability, possibility, and uncertainty. Also, behavior of all systems in the world will change after a period of time. Therefore, researchers are finding new control methods to improve design aspects of control systems. It must be noted that in introduced systems with uncertainty characteristics, definition of fuzzy logic if-then rules includes uncertainty. Today, fuzzy systems are the best choice to transform logic statements (like if-then) into automatic control strategies in order to design controllers for nonlinear systems. However, fuzzy logic systems have some disadvantages, merely. For example, fuzzy rules must be preset by time consuming trial and error methods due to lack of analytical techniques. To solve such a problem, researchers recommend an adaptive fuzzy robust controller using Lyapunov theory. The main idea of using the fuzzy systems with general approximation capability is to model all system uncertainties by linearized parametric uncertainties. Recently, many researches show that type-1 fuzzy systems have some problems and limits in modeling and minimizing of uncertainty effects. Mendel showed that it is wrong to display logic statements by type-1 fuzzy sets (J.M. Mendel. 2007) because statements are uncertain but a type-1 fuzzy set is completely certain and definite. Systems based on logical rules use membership functions (MF). These membership functions are often formulated by a series of pure mathematical functions. In fact, they are not linguistic. The word fuzzy has the connotation of uncertainty but type-1 fuzzy membership function is completely definite when its parameters are certain and it means a paradox (Klir, G. J. et al. 2004). At least four linguistic uncertainties can occur in type-1 fuzzy systems (J.M. Mendel. et al. 2002) because the fuzzy rule-based statements in these systems are pure and certain functions. Basically, there are two types of high level uncertainty: linguistic uncertainty and random uncertainty. Probability theory is used to handle random uncertainty and fuzzy sets are used to handle linguistic uncertainty, and sometimes fuzzy sets can also be used to handle both kinds of uncertainty, because a fuzzy system may use noisy measurements or operate under random disturbances.

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If we use type-1 fuzzy sets to handle random uncertainty, just first order moments of probability density function (pdf) will be used which would not be very useful because random uncertainty requires an understanding of dispersion about the mean, and this information is provided by variance. If fuzzy sets appear in random applications, then both types of uncertainty must be considered. A type-2 fuzzy set has a capability to provide proper estimation of dispersion in uncertain conditions (J.M. Mendel. 2007). Therefore, it is found that a type-2 fuzzy set is capable to handle and minimize the effect of both linguistic and random uncertainties, simultaneously. A wide range of applications in type-2 fuzzy sets show that they provide much better solutions particularly in uncertain conditions (J.M. Mendel. 2007). According to above information and due to presence of uncertainty in fuzzy rules base, it is found that type-2 fuzzy sets provide more robust response against uncertainties compare to type-1 fuzzy sets. In the next section, design of an indirect adaptive fuzzy controller will be presented in which type-2 fuzzy system is used to model an uncertain and nonlinear system.

Design of indirect adaptive controller based on Interval Type-2 fuzzy system:
Assume that there is a nonlinear system which can be presented with nth order differential equations as follows:

$$x^{(n)} = f(x, \dot{x}, ...x^{(n-1)}) + g(x, \dot{x}, ...x^{(n-1)})u \tag{1}$$

$$y = x \tag{2}$$

Where f and g are uncertain, unknown, and nonlinear functions. $u \in R, y \in R$ are input and output of the

process, respectively and $X = (x_1, ..., x_n)^T \in R^n$ is a measurable state vector of the system. If

$g(x) \neq 0$, it can be concluded that (1) is controllable. Here, the objective is to design a type-2 fuzzy feedback controller and present an adaptive rule to tune the parameter vector (θ) such that the system output (γ) reaches to desirable output (γ$_m$) as much as possible. Due to design an indirect adaptive fuzzy controller, it is assumed that there is enough knowledge about control systems. Also, it is assumed that there is an accessible set of fuzzy

if-then rules which can describe the input-output behavior of $g(x), f(x)$. These rules can be written as interval type-2 fuzzy rules as follows:

$$If\ x_1\ is\ F_1^r\ and\ ...and\ x_n\ is\ F_n^r\ then\ \hat{f}\ is\ \tilde{E}^r \tag{3}$$

$$If\ x_1\ is\ G_1^s\ and\ ...and\ x_n\ is\ G_n^s\ then\ \hat{g}\ is\ \tilde{H}^s \tag{4}$$

Which describe $f(x)$ and $g(x)$ respectively. If nonlinear functions, $f(x)$ and $g(x)$, are specified, then we can

choose the control vector (u) to omit the nonlinear part and design a controller based on linear control theory.

In a specific case, it is assumed that $e = y_m - y, e = (e, \dot{e}, ..., e^{(n-1)})^T$ and $K = (k_n, ..., k_1)^T$

K is determined in such a way that all roots of characteristic equation (characteristic polynomial) lie in the left-half S plane (left hand side of imaginary axis). Then, we can select the control rules as follows:

$$u^* = \frac{1}{g(x)}[-f(x) + y_m(n) + K_e^T] \tag{5}$$

The closed-loop system dynamic is obtained by substituting (5) into (1), as:

$$e^{(n)} + k_1 e^{(n-1)} + ...+ k_n e = 0 \tag{6}$$

If the value of K is selected properly, then $\lim_{t \to \infty} e(t) \to 0$. It means that the system's output converges to the desirable output, asymptotically. Equation (5) which is related to ideal controller cannot be used if $f(x)$ and $g(x)$ are unknown. Under these circumstances, only fuzzy if-then rules can be used to describe input-

output behavior of $f(x)$ and $g(x)$ ((3), (4)). Therefore, a reasonable idea is to replace $f(x)$ and $g(x)$ in (5) by fuzzy functions, $\hat{f}(x)$ and $\hat{g}(x)$, which have been obtained from (3) and (4), respectively. Equations (3) and (4) just provide approximate information about $f(x)$ and $g(x)$ functions. Therefore, the fuzzy functions, $\hat{f}(x)$ and $\hat{g}(x)$, are not accurate enough to estimate $f(x)$ and $g(x)$. To improve the accuracy, it is recommended to release some parameters which change online during the operation; in such a way that approximation accuracy improves after a period of time. Assume that $\theta_g \in R^{M_g}$ and $\theta_f \in R^{M_f}$ are free parameters in $\hat{g}(x)$ and $\hat{f}(x)$ functions, respectively. Hence, we have $\hat{g}(x) = \hat{g}(x, \theta)$ and $\hat{f}(x) = \hat{f}(x, \theta)$. In equation (5), by substituting $\hat{g}(x)$ and $\hat{f}(x)$ with $\hat{g}(x, \theta)$ and $\hat{f}(x, \theta)$ the fuzzy controller can be presented as:

$$u = u_I = \frac{1}{\hat{g}(X, \theta)}[-\hat{f}(X, \theta) + y_m^{(n)} + k^T e] \tag{7}$$

This type-2 fuzzy controller is called "certainty equivalent". Now consider interval type-2 fuzzy rules base with m rules as:

If x_1 is F_1^i and ... and x_n is F_n^i then y is $\tilde{G}^r$, $i = 1, ..., m$ $\tag{8}$

With considering product t-norm for combination of primary sets and after applying single fuzzy output, according to primary rules of fuzzy rules base which are related to designed fuzzy system, firing level is defined as:

$$F^i = \prod_{j=1}^{n} \mu_{\tilde{F}_j^i}(x_j) \quad , \quad i = 1, ..., m \tag{9}$$

Because of applying type-2 fuzzy membership functions in fuzzy rules base, after applying single fuzzy input, will be an interval type-1 fuzzy set. Then, (9) can be updated as follows:

$$F^i(x) = [\underline{f}^i(x), \overline{f}^i(x)] \tag{10}$$

Where, $\underline{f}^i(x)$ are $\overline{f}^i(x)$ defined as:

$$\underline{f}^i(x) = \prod_{j=1}^{n} \underline{\mu}_{\tilde{F}_j^i}(x_j) \tag{11}$$

$$\overline{f}^i(x) = \prod_{j=1}^{n} \overline{\mu}_{\tilde{F}_j^i}(x_j) \tag{12}$$

$\underline{\mu}_{\tilde{F}_j^i}(x_j)$ and $\overline{\mu}_{\tilde{F}_j^i}(x_j)$ are the lower and upper bounding membership functions of $\mu_{\tilde{F}_j^i}(x_j)$ respectively.

The next step is the calculation of firing level corresponding to each rule. With considering product t-norm to calculate logic implication (entailment) of each rule, firing level corresponding to each rule will be the product of F^i and G^i. Since the obtained output of each rule is a type-2 fuzzy set before its firing, it must be transformed to a type-1 fuzzy set before deffuzification. In this paper, we use center-of-set (COS) type-reducer strategy (N. N. Karnik. et al. 1998) to obtain inference engine mapping from type-2 fuzzy rules base during the design of

proposed controller. The center-of-set type-reducer method acts such that in ith rules is replaced by its corresponding centroid which is a type-1 fuzzy set and finally, to compute inference engine mapping, it calculates a mean weight from these centroides where the weight corresponding to the centroid of ith rules will be F^i. Therefore, w^i that is corresponding to the centroid G^i, will be obtained as follows:

$$w^i = F^i(x) = \prod_{j=1}^{n} \mu_{\tilde{F}_j^i}(x_j), i = 1, 2, ..., m \tag{13}$$

If we show the centroid of $\tilde{G}^i$ by Y^i ($Y^i = C_{\tilde{G}^i}$), the inference engine mapping for the rules base of (8) will be as (J.M. Mendel. 2006):

$$Y_{\cos}(X) = \int_{y^1} ... \int_{y^m} \int_{f^1} ... \int_{f^m} 1 \bigg/ \dfrac{\sum_{i=1}^{m} f^i y^i}{\sum_{i=1}^{m} f^i} = [Y_l, Y_r] \, st: \tag{14}$$

$$y^t \in \sup p(Y^t) = \sup p(C_{\tilde{G}^i}), i = 1, ..., m$$

Also, the inference engine mapping to approximate $f(x)$ and $g(x)$ is obtained from (3), (4) as:

$$\hat{f}(X) = \int_{y^1} ... \int_{y^{M_f}} \int_{f^1} ... \int_{f^{M_f}} 1 \bigg/ \dfrac{\sum_{i=1}^{M_f} f^i y^i}{\sum_{i=1}^{M_f} f^i} = [\hat{F}_l, \hat{F}_r] \, st: \tag{15}$$

$$y^i \in \sup p(Y^i) = \sup p(C_{\tilde{E}^i}), i = 1, ..., M_f$$

$$\hat{g}(X) = \int_{y^1} ... \int_{y^{M_g}} \int_{g^1} ... \int_{g^{M_g}} 1 \bigg/ \dfrac{\sum_{i=1}^{M_g} g^i y^i}{\sum_{i=1}^{M_g} g^i} = [\hat{G}_l, \hat{G}_r] \, st: \tag{16}$$

$$y^i \in \sup p(Y^i) = \sup p(C_{\tilde{H}^i}), i = 1, ..., M_g$$

Above intervals must be calculated by Kernik-Mendel (KM) algorithm. Now, assume that $y^i (i = 1, ..., M_f)$ and ($i = 1, ..., M_g$) are free parameters gathered in $\theta_f \in R^{M_f}$ and $\theta_g \in R^{M_g}$ respectively. Now, we can rewrite (15) and (16) as:

$$\hat{f}(X, \theta_f) = \theta_f^T \xi(X) =$$

$$\int_{\theta_f^1} ... \int_{\theta_f^{M_f}} \int_{f^1} ... \int_{f^{M_f}} 1 \bigg/ \dfrac{\sum_{i=1}^{M_f} f^i \theta_f^i}{\sum_{i=1}^{M_f} f^i} \tag{17}$$

$$\hat{g}(X,\theta_g) = \theta_g^T \mu(X) =$$

$$\int_{\theta_g^1} \cdots \int_{\theta_g^{M_g}} \int_{g^1} \cdots \int_{g^{M_g}} \Bigg/ \frac{\sum_{i=1}^{M_g} g^i \theta_g^i}{\sum_{i=1}^{M_g} g^i} \tag{18}$$

Where $\xi(X)$ and $\mu(X)$ are M_f and M_g dimensional vectors, respectively and their ith elements are calculated as:

$$\xi^i(x) = \frac{f^i}{\sum_{k=1}^{M_f} f^k} \qquad i = 1,....,M_f \tag{19}$$

$$\eta^i(x) = \frac{g^i}{\sum_{k=1}^{M_g} g^k} \qquad i = 1,....,M_g \tag{20}$$

After above calculations, two interval type-1 fuzzy sets, $\hat{g}(X,\theta_g) = [\hat{G}_l, \hat{G}_r]$ and $\hat{f}(X,\theta_f) = [\hat{F}_l, \hat{F}_r]$, are obtained.

Since f and g are interval type-2 fuzzy sets, defuzzification stage (J.M. Mendel. 2006) will be as:

$$\hat{f}(X,\theta_f) = \frac{\bar{F}_l + \bar{F}_r}{2}, \hat{g}(X,\theta_g) = \frac{\bar{G}_l + \bar{G}_r}{2} \tag{21}$$

According to (J.M. Mendel. 2006), $\hat{F}_l, \hat{F}_r, \hat{G}_l, \hat{G}_r$ can be written as:

$$\hat{F}_l = \theta_f^T \xi_l(X) \quad , \quad \hat{F}_r = \theta_f^T \xi_r(X) \tag{22}$$

$$\hat{G}_l = \theta_g^T \eta_l(X) \quad , \quad \hat{G}_r = \theta_g^T \eta_r(X) \tag{23}$$

Where $\xi_l(X)$ and $\xi_r(X)$ are M_f - dimensional vectors whose ith elements are obtained from:

$$\xi_l^i(X) = \frac{f_l^i}{\sum_{j=1}^{M_f} f_l^j}, \xi_r^i(X) = \frac{f_r^i}{\sum_{j=1}^{M_f} f_r^j} \tag{24}$$

Similarly, $\eta_r(X)$ and $\eta_l(X)$ are M_g- dimensional vectors whose ith elements are calculated from:

$$\eta_l^i(X) = \frac{g_l^i}{\sum_{j=1}^{M_g} g_l^j}, \eta_r^i(X) = \frac{g_r^i}{\sum_{j=1}^{M_g} g_r^j} \tag{25}$$

The following equations are obtained by substituting (22) and (23) into (21):

$$\hat{f}(X,\theta_f) = \frac{1}{2}\theta_f^T(\xi_l(X) + \xi_r(X)) \tag{26}$$

$$\hat{g}(X,\theta_g) = \frac{1}{2}\theta_g^T(\eta_l(X) + \eta_r(X)) \tag{27}$$

By defining parameters such as (Li-xin Wang. 2002) and considering positive definite Lyapunov function as (28),

$$V = \frac{1}{2}e^T Pe + \frac{1}{2\gamma_1}(\theta_f - \theta_f^*)^T(\theta_f - \theta_f^*) + ...$$
$$\frac{1}{2\gamma_2}(\theta_g - \theta_g^*)^T(\theta_g - \theta_g^*) \tag{28}$$

voltage-time derivative ($\dot{V}$) through closed-loop path (6) is obtained as follows:

$$\dot{V} = -\frac{1}{2}e^T Pe + e^T Pbw...$$
$$+\frac{1}{\gamma_1}(\theta_f - \theta_f^*)^T[\dot{\theta}_f + \frac{1}{2}\gamma_1 e^T Pb(\xi_l(X) + \xi_l(X))] +$$
$$\frac{1}{\gamma_2}(\theta_g - \theta_g^*)^T[\dot{\theta}_g + \frac{1}{2}\gamma_1 e^T Pb(\eta_l(X) + \eta_l(X))u_I] \tag{29}$$

To minimize tracking error (*e*), adaptation rules must be chosen such that $\dot{V}$ becomes negative definite.

$\frac{1}{2}e^T Pe$ is a negative term and we are able to choose fuzzy systems in such a way that minimum approximation

error (*w*) becomes small. Therefore, a good strategy is to select the adjustment rule such that the last two terms of (29) become zero. Hence, the adaptation rules can be written as follows:

$$\dot{\theta_f} = \frac{-\gamma_1}{2}e^T pb(\xi_l(X) + \xi_r(X)) \tag{30}$$

$$\dot{\theta_g} = \frac{-\gamma_2}{2}e^T pb(\eta_l(X) + \eta_r(X))u_I \tag{31}$$

Indirect adaptive type-2 fuzzy control system has been shown in Fig. (1), briefly.

Simulation Results:
In this section, we apply an indirect adaptive fuzzy tracking controller to the inverted pendulum model shown

in Fig. (2). The objective is that pendulum angular position tracks desirable response ($y_m(t) = \frac{\pi}{30}\sin(t)$) as

much as possible. The dynamic relations have been presented in (Li-xin Wang. 2002) (section 20.2.2) where

is angular acceleration, x_2 is angular velocity, $y = x_1$ is angular position of the rod, and u is applied force

(control vector). Parameters of the model have been chosen as presented in (Li-xin Wang. 2002), section 23.3. Simulation results without any uncertainty in the model parameters (the length and mass of the rod) have been presented in table (1) for both type-1 and type-2 fuzzy sets. (The initial conditions are
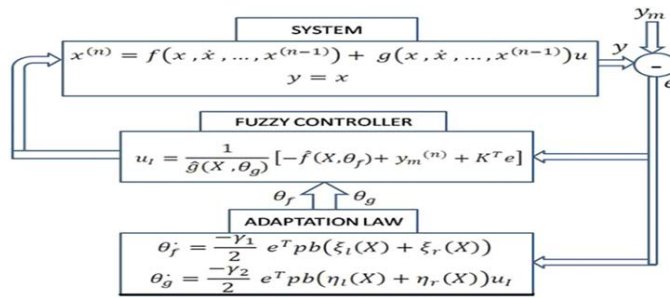
Fig. 1: Indirect adaptive type-2 fuzzy control system

Table 1: Results without any uncertainty and comparison with SSE index

Initial Condition	Fuzzy Type 1	Fuzzy Type 2
$X(0) = (-\frac{\pi}{60}, 0)^T$	SSE=0.1090	SSE= 0.0852
$X(0) = (\frac{\pi}{60}, 0)^T$	SSE=0.0990	SSE= 0.0726

$X(0) = (\frac{\pi}{60}, 0)^T$ and $X(0) = (-\frac{\pi}{60}, 0)^T$. Interval adaptive type-2 fuzzy controller

rules base has been considered in table (1) as a form of membership functions shown in Fig. (3).
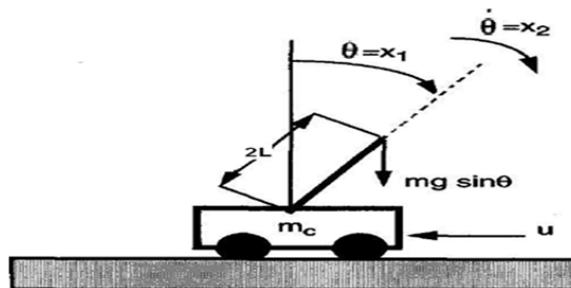

Fig. 2: Inverted pendulum model

the comparison index in this table is sum of the squared error (SSE) between y and y_m during an interval of

[0, 20] s. The membership functions in Fig. (3) are the same as presented in (Li-xin Wang. 2002).
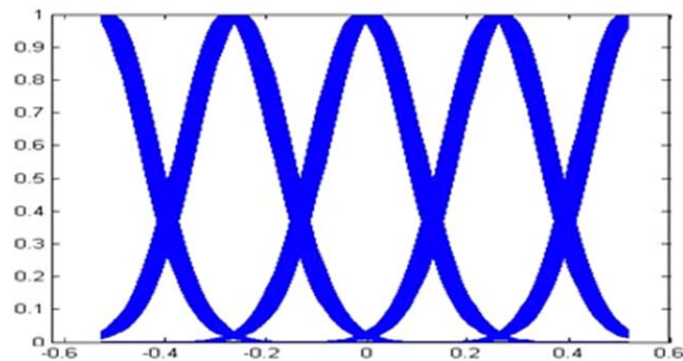

Fig. 3: Interval Type-2 fuzzy membership functions (with variable standard deviation)

The only difference is that a tolerance ($-0.02 \leq \Delta \leq 0.02$) is applied to the mean of these functions in order to make them uncertain.

The simulation results with the presence of uncertainty in the parameters of the model have been shown in tables (2) and (3). It must be noted that uncertainty is applied to the length and mass of the rod, separately and a random noise with normal distribution around zero point, is applied to the length and mass of the rod in scale of 0.05 and 0.01, respectively.

Tables (4), (5), and (6) are similar to (1), (2), and (3) but the tolerance is between -0.04 and 0.04. Here, just we compare two cases of tracking signal performance between type-1 and type-2 fuzzy systems. In figures (4) and (5), the tracking control performance is shown for type-1 and type-2 fuzzy systems respectively when the system model is certain and without any uncertainty.

In figures (6) and (7), the tracking control performance is shown for type-1 and type-2 fuzzy systems respectively when the system model is uncertain. (The initial condition is and uncertainty is applied to the length of the rod). It must be noted that membership functions in all simulations of this paper, include variable standard deviation because such a membership functions are able to model the behavior of the system much better.

Table 2: Results with the presence of uncertainty in the length

Initial Condition	Fuzzy Type 1	Fuzzy Type 2
$X(0) = (-\dfrac{\pi}{60}, 0)^T$	SSE=1.3682	SSE= 1.2011
$X(0) = (\dfrac{\pi}{60}, 0)^T$	SSE=1.3189	SSE= 1.1702

Table 3: Results with the presence of uncertainty in the mass

Initial Condition	Fuzzy Type 1	Fuzzy Type 2
$X(0) = (-\dfrac{\pi}{60}, 0)^T$	SSE=1.2704	SSE= 1.1501
$X(0) = (\dfrac{\pi}{60}, 0)^T$	SSE=1.2313	SSE= 1.1090

Table 4: Results without any uncertainty and comparison with SSE index (with new)

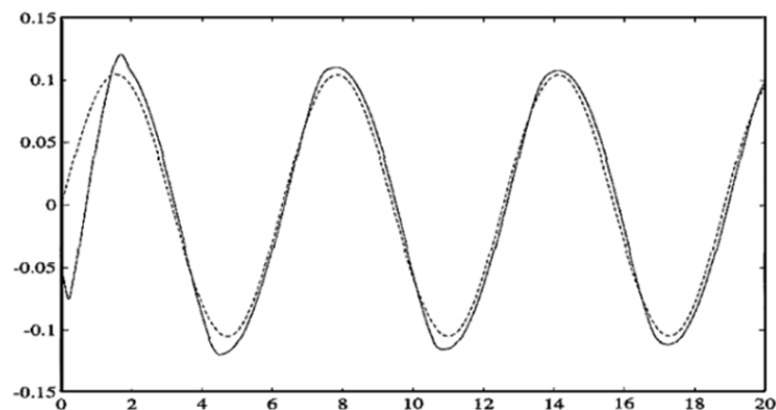| Initial Condition	Fuzzy Type 1	Fuzzy Type 2
$X(0) = (-\dfrac{\pi}{60}, 0)^T$	SSE=0.1090	SSE= 0.0730
$X(0) = (\dfrac{\pi}{60}, 0)^T$	SSE=0.0990	SSE= 0.0603

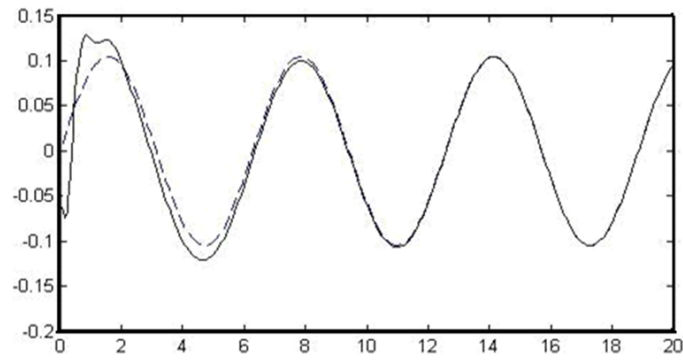**Fig. 4:** Tracking performance with Type-1 fuzzy set

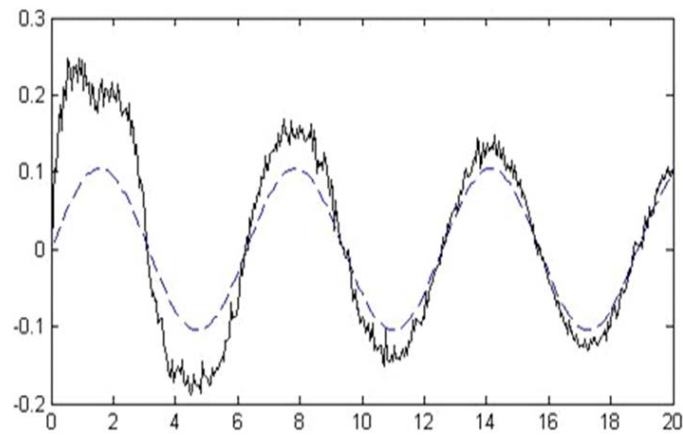**Fig. 5:** Tracking performance with proposed Type-2 fuzzy set

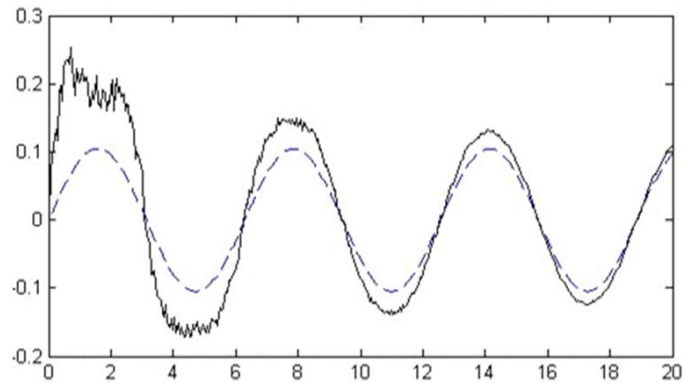**Fig. 6:** Tracking performance with Type-1 fuzzy set (uncertainty in the length)

Fig. 7: Tracking performance with proposed Type-2 fuzzy set (uncertainty in the length)

Table 5: Results with the presence of uncertainty in the length (with new)

Initial Condition	Fuzzy Type 1	Fuzzy Type 2
$X(0) = (-\frac{\pi}{60}, 0)^T$	SSE=1.3682	SSE= 1.1032
$X(0) = (\frac{\pi}{60}, 0)^T$	SSE=1.3189	SSE= 1.0508

Table 6: Results with the presence of uncertainty in the mass (with new)

Initial Condition	Fuzzy Type 1	Fuzzy Type 2
$X(0) = (-\dfrac{\pi}{60}, 0)^T$	SSE=1.2704	SSE= 1.0203
$X(0) = (\dfrac{\pi}{60}, 0)^T$	SSE=1.2313	SSE= 1.0063

Conclusion:

It can be concluded from simulation results that proposed controller (type-2 fuzzy controller) has a better tracking performance and robustness compare to type-1 fuzzy controller, in both certain and uncertain model of the system. The reason is that type-2 fuzzy system has a better performance in uncertain conditions rather than type-1 fuzzy system. Also, it can be found that membership functions with more uncertainty, lead to handle system uncertainty conditions much better and therefore, effect of uncertainties will be minimized. Also, it must be noted that more uncertain membership function does not lead to less uncertainty condition. But, it is very important to choose an optimal value Δ according to each problem. In addition, an optimal selection for Δ is a nonlinear optimization problem, solely. Finally, following new ideas are proposed to accomplish researches about the fuzzy systems and controller design:

1) Using Type-1 and Type-2 fuzzy sets to apply inputs.
2) Determination of optimal value Δ in control problems of inverted pendulum.
3) Type-2 fuzzy tracking control in the presence of noise measurements.
4) Using other type-reduction methods and compare them

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