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Using Type-2 Fuzzy Methods in the Design of a Real -Time Robust Regulator around operating point of the nonlinear system

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Abstract: This paper presents tow type-2 fuzzy robust regulators for the nonlinear and uncertain systems. 1-based on type-2 fuzzy Adaptive controller(non real-time) & 2-based on linearized model of system and jacobian matrix and a new type-2 fuzzy inference engine(real time). Recently , we present the First method in (Chahkandi, *et al*., 2011) now we want to present secondary method. first, state equations of the nonlinear system is linearized around operating point and then a fuzzy system is designed based on state vector of the linearized system and knowledge about the control system. The designed fuzzy system uses control signals to regulate outputs through measurements obtained from system state vector and basic fuzzy rules of the human knowledge. In this paper, to immediate use of the regulator, type-2 fuzzy system is designed based on uncertainty bounds technique. Finally, simulations are implemented for an individual inverted pendulum model in two cases(for both methods): certain and uncertain equations of the system. In both cases, the simulation results show that proposed regulators(1&2) has a better performance in convergence speed and robustness compare to type-1 fuzzy regulators. Finally, two methods have been compared together in three indexes: 1.ease of implementation, 2. convergence speed and robustness & 3. Speed of Computational

Key words: Interval type-2 fuzzy system; Jacobian matrix; Robust regulator; Uncertainty bounds; T1: Type-1; T2: Type-2; FS: Fuzzy Set.

INTRODUCTION

The control of nonlinear systems has been an important research topic Slotine and Li, (1991). Traditionally, control system design has been tackled using mathematical models derived from physical laws.but In fact, most of the parameters and structures of the system are unknown due to environment changes, modelling errors and unmodeled dynamics. An overview about such a systems is that they have unknown and uncertain equations. Actually, in addition to unknown mathematical model of the system, they are uncertain too. To overcome the above problems in the design of control systems several techniques have been emerged in the recent years especially techniques based on the intelligent technology such as neural networks, fuzzy logic, genetic algorithms and evolutionary computation . It is obvious that our world is based on probability, possibility, and uncertainty. Also, behavior of all systems in the world will change after a period of time. Therefore, researchers are finding new control methods to improve design aspects of control systems. It must be noted that in introduced systems with uncertainty characteristics, definition of fuzzy logic if-then rules includes uncertainty. In particular, fuzzy logic systems (FLS) have been successfully applied to control complex or ill-defined processes whose mathematical models are difficult to obtain. The ability of converting linguistic descriptions into automatic control strategy has made it a practical and promising alternative to the classical control scheme for achieving control of complex nonlinear systems . Recently, Many researches have shown that type-1 FLS have difficulties in modeling and minimizing the effect of uncertainties. Mendel showed that it is wrong to display words by type-1 fuzzy sets Mendel, (2007) because words are uncertain but a type-1 fuzzy set is completely certain and definite. One reason is that a type-1 fuzzy set is certain in the sense that the membership grade for a particular input is a crisp value. Recently, type-2 fuzzy sets, characterized by membership functions (MF) that are themselves fuzzy, have been attracting interest (Mendel, 2001; Castillo, O. and P. Melin, 2008). A FLS using at least one type-2 fuzzy set is called a type-2 FLS. The word fuzzy has the connotation of uncertainty but type-1 fuzzy membership function is completely definite when its parameters are certain and it means a paradox Klir and Folger, (1988). At least four linguistic uncertainties can occur in type-1 fuzzy systems Mendel and John, (2002) because the fuzzy rule-base in these systems are

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pure and certain functions. Basically, there are two types of high level uncertainty: linguistic uncertainty and random uncertainty. Probability theory is used to handle random uncertainty and fuzzy sets are used to handle linguistic uncertainty, and sometimes fuzzy sets can also be used to handle both kinds of uncertainty, because a fuzzy system may use noisy measurements or operate under random disturbances. If we use type-1 fuzzy sets to handle random uncertainty, just first order moments of probability density function (pdf) will be used which would not be very useful because random uncertainty requires an understanding of dispersion about the mean, and this information is provided by variance. If fuzzy sets appear in random applications, then both types of uncertainty must be considered. A type-2 fuzzy set has a capability to provide proper estimation of dispersion in uncertain conditions Mendel, (2007). Hence, it is found that a type-2 fuzzy set is capable to handle and minimize the effect of both linguistic and random uncertainties, simultaneously. A wide range of applications in type-2 fuzzy sets show that they provide much better solutions particularly in uncertain conditions Mendel, (2007). A set of applications and papers about type-2 fuzzy sets have been presented in (www.type2fuzzylogic.org; Castillo and Melin, 2008). Similar to the conventional adaptive control, adaptive fuzzy control can be categorized into direct, indirect and composite schemes according to the type of fuzzy rules. According to above information and due to existing uncertainty in fuzzy rules, it is found that type-2 fuzzy sets provide more robust response compare to type-1 fuzzy sets. In the next section, a type-2 fuzzy regulator will be designed to provide an appropriate control signal for output regulation of nonlinear and uncertain systems. The regulator has been designed to regulate a nonlinear system in an individual operating point.

II. Design of type-2 fuzzy robust regulator based on uncertainty bounds technique (Method 2):

Dynamic of most industrial processes and real systems is nonlinear. So, analysis and design of control systems in such cases is very difficult and application of nonlinear controllers in most practical cases is not necessary. Actually, experiments show that the linear control systems cover a wide control range of real systems and complicated industrial processes. So, it is very important and unavoidable to obtain accurate linear model from nonlinear systems in engineering sciences. A practical method in linearization of nonlinear equations is to use Taylor series expansion. According to this method, equations of n-order nonlinear system (i.e., equation (1)) can be linearized around its equilibrium point as equations (2).

$$X = f(X,U,t)$$
$$Y = g(X,U,t) \tag{1}$$

$$X = AX(t) + BU(t)$$
$$Y = CX(t) + DU(t) \tag{2}$$

It is assumed that g and f are nonlinear and uncertain. Also, U and Y are process input and output, respectively. $X = (x_1, \dots, x_n)^T \in R^n$ is a measurable state vector. A,B are Jacobian matrix of f, with respect to X,U, respectively. Also, C, D are Jacobian matrix of g, with respect to X, U, respectively. To calculate these matrixes, refer to Slotine and Li, (1991) and Khaki Seddigh, (2005). This method shows how to apply linear control systems to practical problems and ensures that stable linear control design can provide a local stability around operating point in the main system Slotine and Li, (1991). By applying these robust control methods on linearized model of the system, it can be guaranteed that system is stable around operating point in spite of process changes. It must be noted that each point in state space can reach to an equilibrium point through change of variables. To design a fuzzy regulator, it is assumed that there is enough knowledge about control systems. Also, it is assumed that there is an accessible set of fuzzy if-then rules which can describe the behavior of control system. According to given explanation about type-2 fuzzy sets, in this paper, fuzzy rules are considered as type-2 to make a robust regulator. Therefore, fuzzy if-then rules are specified as follows:

$$if\ x_1 is\ \tilde{A}_1^i, \dots, x_n is\ \tilde{A}_n^i\ \ then\ U\ is\ \tilde{C}^i\ , i = 1, \dots, m \tag{3}$$

These rules extract control strategies from state vector to regulate system's outputs moment by moment. In fact, system state is reported to tuned-fuzzy inference engine (FIE) and FIE sends the control range of U in order to regulate the output or state. With considering product t-norm for combination of antecedent sets and after applying singleton fuzzy for input sets, according to primar rules of fuzzy rules base which are related to designed fuzzy system, firing level is defined as follows:

$$A^i = \prod_{j=1}^{n} \mu_{\tilde{A}_j^i}(x_j) \quad , \quad i = 1, \dots, m \tag{4}$$

Because of applying type-2 fuzzy membership functions in fuzzy rules base, after applying singleton fuzzy input, A^i will be an interval type-1 fuzzy set which is defined as a range .Then, equation (4) can be updated as follows:

$$A^i(x) = \left[\underline{A}^i(x), \overline{A}^i(x)\right] \tag{5}$$

Where, $\underline{A}^i(x)$ and $\overline{A}^i(x)$ are defined as:

$$\underline{A}^i(x) = \prod_{j=1}^{n} \overline{\mu}_{\tilde{A}_j^i}(x_j) \tag{6}$$

$$\overline{A}^i(x) = \prod_{j=1}^{n} \underline{\mu}_{\tilde{A}_j^i}(x_j) \tag{7}$$

Also, $\underline{\mu}_{\tilde{A}_j^i}(x_j)$ and $\overline{\mu}_{\tilde{A}_j^i}(x_j)$ are membership functions as a lower and upper bound for $\mu_{\tilde{A}_j^i}(x_j)$. The next step is the calculation of centroid of consequent fuzzy set which is proportional to each rule. Centroid of consequent set for each rule is defined as:

$\tilde{C}^i = [\, \tilde{C}^i{}_l \, , \, \tilde{C}^i{}_r]$ which is an interval type-1 fuzzy set Mendel, (2001). Within regulator design,uncertainty bounds theory is used to obtain mapping of fuzzy inference engine Wu and Mendel, (2002). In fact, system based on uncertainty bounds theory consists of two parallel processes obtained from type- 1 fuzzy calculations which can be operated as a real time. Since type-reduction methods Karnik and Mendel, (1998) are not used in this strategy, this inference speed is almost as equal as type-1 fuzzy inference engine. Therefore, this system is used more than type-reduction methods specially in online and immediate applications Mendel, (2001) because type-reduction methods require using KM (Karnik-Mendel) iteration algorithm Karnik and Mendel, (2001). The calculations of uncertainty bounds-based inference engine have been shown in Fig. (1) where, is an approximation of control signal. To calculating $\underline{U}_l , \overline{U}_l , \underline{U}_r , \overline{U}_r$ must be determined. For more information about this method, refer to Wu and Mendel, (2002). The designed regulator has been shown in Fig. (2), briefly. It must be noted that system state and appropriate control signal are input and output of type-2 fuzzy system, respectively.

III. Simulation Results:
For method 2:

In this section, simulation results will be investigated and Comparison between type-1 and type-2 fuzzy sets to regulate inverted pendulum on unstable equilibrium point, will be implemented. To achieve this purpose, we apply fuzzy regulator to an inverted pendulum shown in Fig. (3).

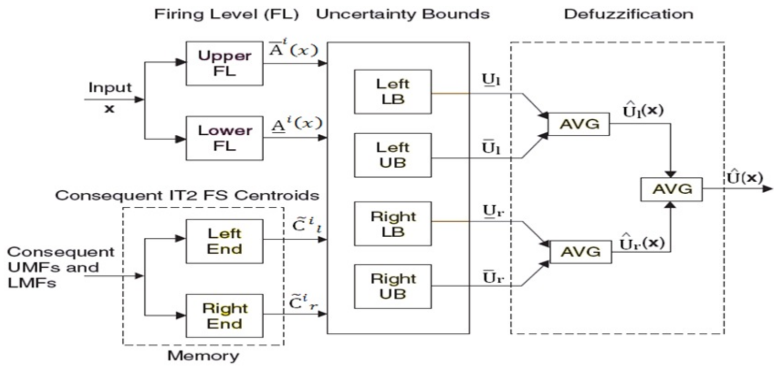

Fig. 1: Calculation of an Interval T2 Fuzzy set using uncertainty bounds technique instead of type-reduction methods

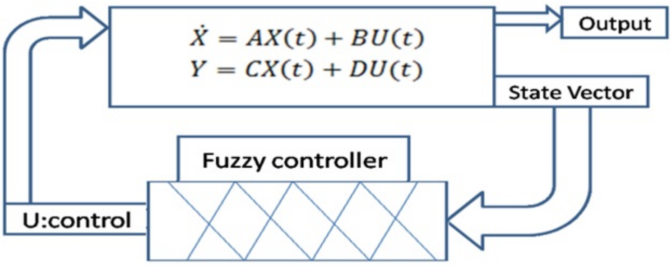

Fig. 2: Fuzzy regulator

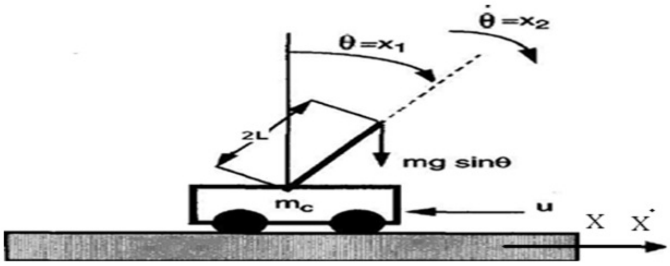

Fig. 3: Inverted pendulum system

the control purpose is to regulate the angular position of pendulum on its unstable equilibrium point independent of cart's position. $\dot{\theta}, \theta$ are the inputs of fuzzy system and restoring force is considered as output. Nonlinear and linearized dynamic equations of the system around point (0, 0) have been presented in Khaki Seddigh, (2005), section 1-2-2. Parameters of the model are considered as follows:

$m_c = 2Kg, m = 0.1Kg, l = 0.5, g = 10m/s^2$

Due to linearized model of the system, initialcondition of pendulum must have a very small deviation from its vertical position (i.e., operating point). Otherwise, linearized model would not be valid. Table (1) shows the simulation results for type-1 and type-2 fuzzy sets without any uncertainty in parameters. (The parameters are length and mass of the rod). Initial conditions are: $X(0) = [\theta = 0.1, \dot{\theta} = 0.2, x = 0, \dot{x} = 0]$. Rule-based type-2 fuzzy membership functions which are considered in table (1), have been shown in Figures (4), (5) and (6). Comparison index in this table is considered as Mean-squared Error between desirable pendulum path and regulated pendulum path which takes values in the interval [0, 10] sec.

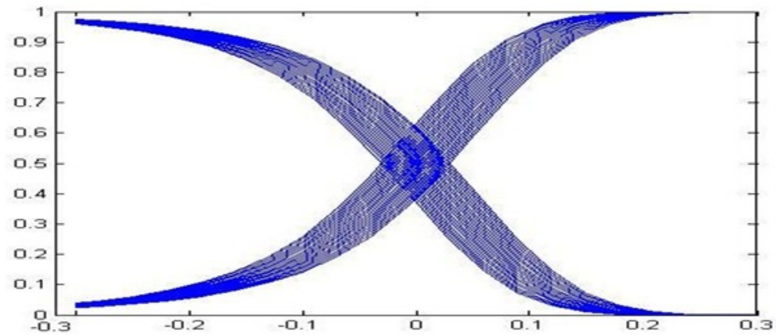

Fig. 4: T2 fuzzy membership functions defined for

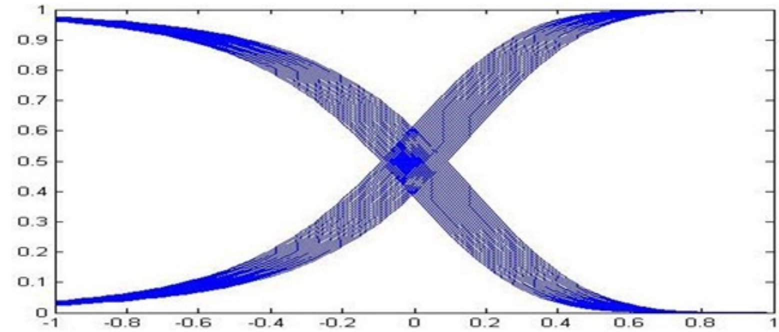

Fig. 5: T2 fuzzy membership functions defined for

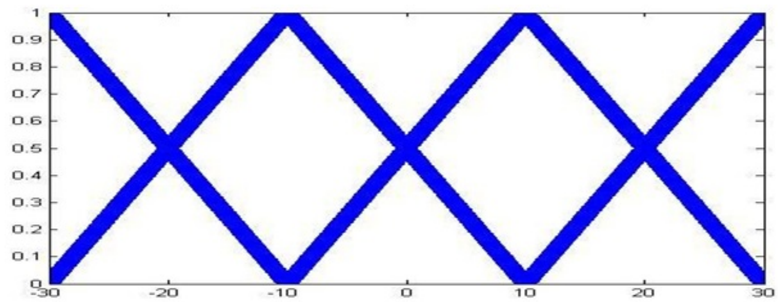

Fig. 6: T2 fuzzy membership functions defined for control vector U

The membership functions as shown in Figures 4-6, are the same as presented in Mokhtari and Marie, (2000), section 3, part 4-1. The only difference is a tolerance (ε) which is applied to center of these functions in order to make them uncertain. Also, table (2) shows the simulation results while the parameters of the model are indefinite. It must be noted that uncertainty is applied to the length and mass of the rod, separately and a random noise with normal distribution around zero point, is applied to the length and mass of the rod in scale of 0.05 and 0.01, respectively. MSE index has been separately inserted in table (2) for the length and mass of the rod under uncertain condition. Finally, to examine the regulator performance, a comparison between type-1 and type-2 fuzzy system has been implemented in figures (7) – (9). The performance of type-1 and type-2 fuzzy regulator has been shown in Fig. (7), under certain condition. The performance of type-1 and type- 2 fuzzy regulator has been shown in figures (8) and (9), respectively under uncertain condition in the length of the rod.

Table 1: Comparison between T1 and T2 FS under certain condition

Control system modeling	Type-1 Fuzzy Set	Type-2 Fuzzy Set
(MSE)	19.47e-005	1.399e-005

Table 2: Comparison between T1 and T2 FS under uncertain condition

Control system modeling	Type-1 Fuzzy Set	Type-2 Fuzzy Set
Uncertainty in length (MSE)	36.17e-005	2.210e-005
Uncertainty in mass (MSE)	33.82e-005	2.032e-005

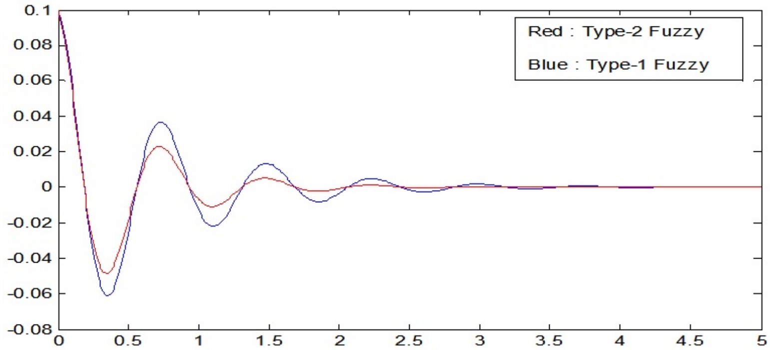

Fig.7: Comparison between T1 and T2 fuzzy regulator (certain process model)

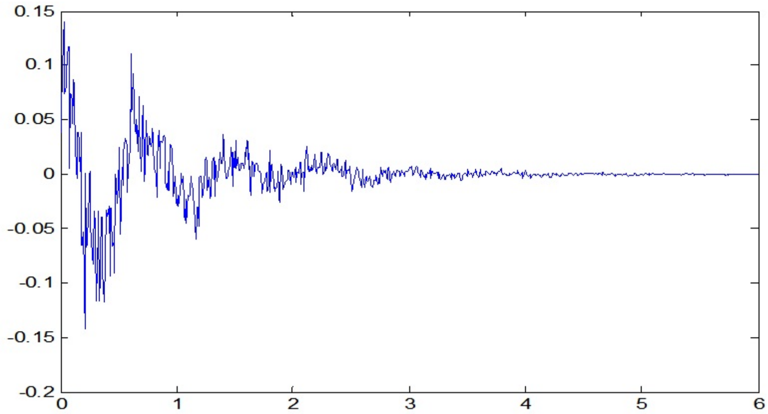

Fig. 8: Performance of T1 fuzzy regulator (uncertainty in length)

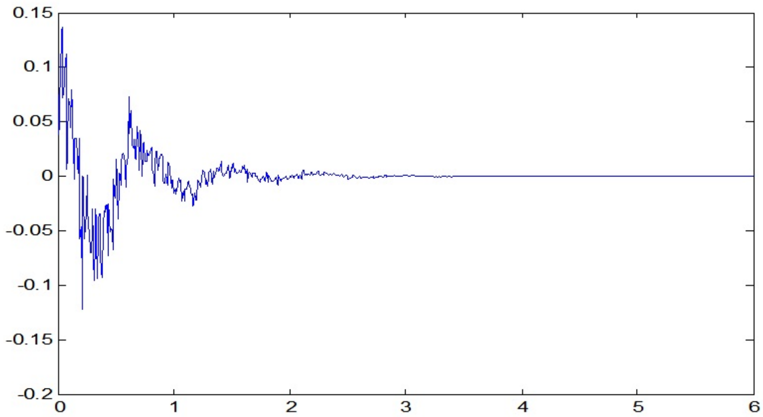

Fig. 9: Performance of proposed T2 fuzzy regulator (Uncertainty in length)

For method 1:

In this section, we apply an indirect adaptive fuzzy tracking controller to the inverted pendulum model shown in Fig. (3). The objective is that pendulum angular position tracks desirable response (y_m=0) as much as possible. The dynamic relations have been presented in Li-xin Wang, (1997) (section 20.2.2). all process of design for this controller have been presented in Chahkandi Nejad, *et al.,* (2011). Indirect adaptive type-2 fuzzy controller has been shown in Fig. (10), briefly.

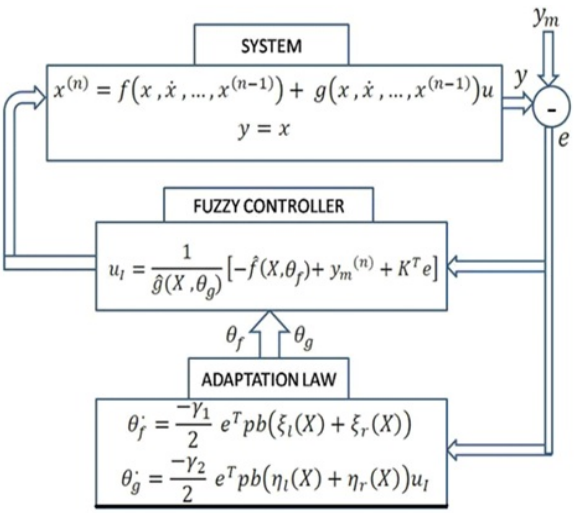

Fig. 10: Indirect adaptive type-2 fuzzy control system

Briefly, Table (3) and Table (4) show the simulation results for type-1 and type-2 fuzzy adaptive controller without any uncertainty and with uncertainty in parameters, respectively..Rule-based type-2 fuzzy membership functions which are considered in table (1) and (2), have been shown in Figure (11) (for). Comparison index in this table is considered as Mean-squared Error between desirable pendulum path and regulated pendulum path which takes values in the interval [0, 10] sec.

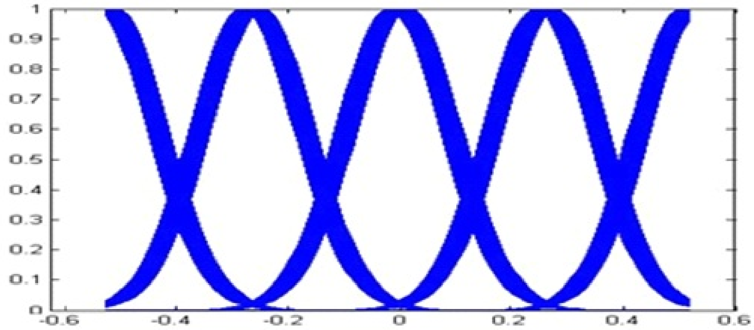

Fig. 11: Interval Type-2 fuzzy membership functions (with variable standard deviation)

The membership functions in Fig. (3) are the same as presented in (Li-xin Wang. 2002). The only difference is that a tolerance (-0.02<<0.02) is applied to the mean of these functions in order to make them uncertain.

Table 3: Results without any uncertainty and comparison with SSE index

Control system modeling	Type-1 Fuzzy Set	Type-2 Fuzzy Set
(MSE)	11.47e-005	0.599e-005

Table 4: Comparison between T1 and T2 FS under uncertain condition

Control system modeling	Type-1 Fuzzy Set	Type-2 Fuzzy Set
Uncertainty in length (MSE)	28.17e-005	1.410e-005
Uncertainty in mass (MSE)	25.82e-005	1.232e-005

Finally, to examine the regulator performance, a comparison between type-1 and type-2 fuzzy system has been implemented in figures (12) – (14). The performance of type-1 and type-2 fuzzy regulator has been shown in Fig. (12), under certain condition. The performance of type-1 and type- 2 fuzzy regulator has been shown in figures (13) and (14), respectively under uncertain condition in the length of the rod.

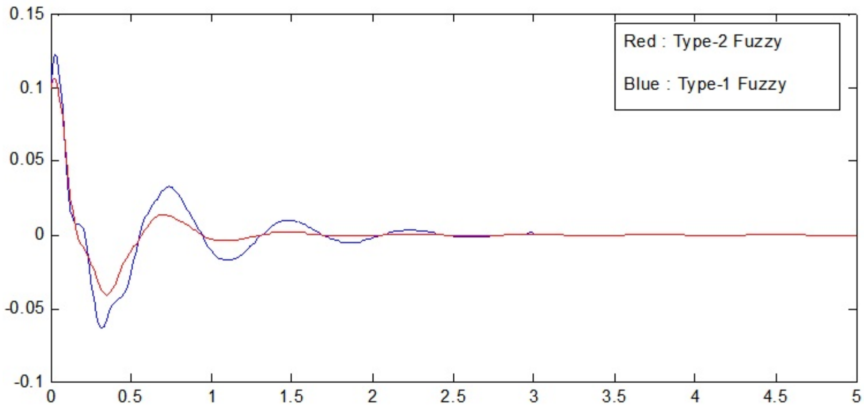

Fig. 12: Comparison between T1 and T2 fuzzy Adaptive controller (certain process model)

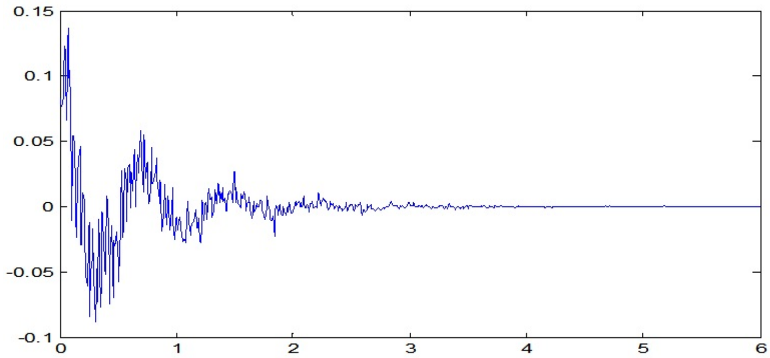

Fig. 13: Performance of T1 fuzzy adaptive controller (uncertainty in length)

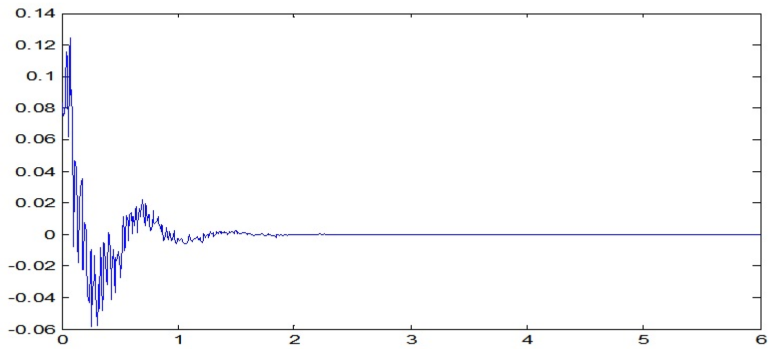

Fig. 14: Performance of proposed T2 fuzzy Adaptive controller (Uncertainty in length)

Table 5: Two methods have been compared together in three indexes : 1.ease of implementation, 2. convergence speed and robustness & 3. Speed of Computational

Comparison index	ease of implementation	Robustness and convergence speed	Speed of Computational
Regulator based on type-2 fuzzy Adaptive controller (non real time method)	Poor	Very Good	Poor
Regulator based on linearized model of system (real time method)	Good	Good	Very Good

Conclusion:

It can be concluded from simulation results that proposed regulators (type-2 fuzzy regulators method 1&2) have a better convergence speed and robustness compare to similar type-1 fuzzy regulators, in both certain and uncertain model of the system. The reason is that type-2 fuzzy system has a better performance in uncertain conditions rather than type-1 fuzzy system. Also, by applying uncertain type-2 membership functions, uncertainty of the model can be handled much better than before. It must be noted that more uncertain membership function does not lead to less uncertainty condition. But, it is very important to choose an optimal value according to each problem. In addition, an optimal selection for is an optimization problem, solely. Finally, it can be concluded that by applying a type-2 fuzzy regulator, the system can achieve to more robust performance around operating point. Also, following new ideas are proposed to accomplish researches about the fuzzy systems and regulator design:

1) Applying type-1 and type-2 fuzzification in order to reduce noises in measurements.
2) Determination of optimal value of ($\varepsilon\&\Delta$) in control problems of inverted pendulum.
3) Increase of convergence zone around operating point by applying optimal type-2 fuzzy membership functions.

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