Australian Journal of Basic and Applied Sciences, 5(4): 37-43, 2011
ISSN 1991-8178

MHD Viscoelastic Fluid Towards Stagnation Point on a Vertical Surface

[1]Kartini Ahmad and [2]Roslinda Nazar

[1]Department of Mathematics, Centre for Foundation Studies, International Islamic University Malaysia, 46350 Petaling Jaya, Selangor, Malaysia
[2]School of Mathematical Sciences, Faculty of Science & Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia

Abstract: The steady two-dimensional magnetohydrodynamic (MHD) mixed convection flow near the stagnation point of a vertical surface in a viscoelastic fluid is investigated in this paper. The temperature of the wall is assumed to vary linearly with the distance from the stagnation point. The partial differential equations which governed the flow and thermal fields are transformed into a system of ordinary differential equations, which are then solved numerically using an implicit finite-difference scheme known as the Keller-box method. The numerical results for the local Nusselt number Nu_x and the skin friction coefficient C_f are obtained and discussed in detail for various physical parameters such as the magnetic parameter M, viscoelastic parameter K, Prandtl number Pr and mixed convection parameter l for both assisting ($\lambda > 0$) and opposing ($\lambda < 0$) flows. The numerical values obtained are shown in tables and features of the flow and heat transfer are presented in the form of graphs.

Key words: boundary layer; MHD; mixed convection; stagnation point; viscoelastic fluid

INTRODUCTION

The theory of non-Newtonian fluids has been actively discussed over the last few decades due to its importance in industrial applications. Pioneering works by Oldroyd [1], Beard and Walters [2] and Rajagopal et al. [3] who have developed the boundary layer theory for second-grade fluids, namely viscoelastic fluid which exhibits both elastic and viscous properties have motivated many researchers to really explore this kind of fluid with various situations.

The study of a non-Newtonian fluid flow in the region of stagnation point has been done by several authors, such as Srivastava [4], Rajeswari and Rathna [5], Beard and Walters [6], Garg and Rajagopal [7], Ariel [8], Mahapatra and Gupta [9] and very recently by Ayub et al. [10] who studied the stagnation point flow of viscoelastic fluid towards a stretching sheet and Li et al. [11] for the case of oblique stagnation point flow of a viscoelastic fluid with the effect of heat transfer. Further, the study of mixed convection in the stagnation point flow in a viscoelastic fluid has been done by Ramachandran et al. [12] for viscous fluid, while Hayat et al. [13] and Anwar et al. [14] considered the flow of a viscoelastic fluid over a vertical surface and horizontal circular cylinder, respectively.

However, problems with associated MHD have not received considerable attention until recently. Ishak et al. [15] considered MHD flow towards a stagnation point on a vertical surface for micropolar fluid. Mahapatra et al. [16] studied two-dimensional steady stagnation-point flow of an incompressible viscoelastic fluid over a flat deformable surface when the surface is stretched in its own plane with a velocity proportional to the distance from the stagnation-point, and very recently, Prasad et al. [17] performed an analysis of flow and heat transfer characteristics in an incompressible electrically conducting non-Newtonian boundary layer flow of a viscoelastic fluid over a stretching sheet, to name just a few.

The present problem is an extension of the problem considered in the paper of Hayat et al. [13], by considering steady two-dimensional MHD mixed convection flow towards a stagnation point on a vertical surface immersed in a viscoelastic fluid.

Basic Equations:

Consider a steady mixed convection boundary layer flow of an incompressible electrically conducting viscoelastic fluid over a semi-infinite vertical surface, which is placed in such fluid with uniform ambient

Corresponding Author: Kartini Ahmad, Department of Mathematics, Centre for Foundation Studies, International Islamic University Malaysia, 46350 Petaling Jaya, Selangor, Malaysia
E-mail: katzdany@yahoo.com

temperature T_∞. The velocity of the flow external to the boundary layer is $U(x)$ and the temperature $T_w(x)$ of the plate are proportional to the distance x from the stagnation point, i.e. $U(x)=ax$ and $T_w(x)=T_\infty+bx$, where a and b are constant. A uniform magnetic field of strength B_o is applied in the positive y-direction. It is assumed that the surface of the plate is heated or cooled to a variable $T_w(x)$, where $T_w(x) > T_\infty$ is for a heated plate and $T_w(x) < T_\infty$ is for a cooled plate, as shown in Fig. 1.

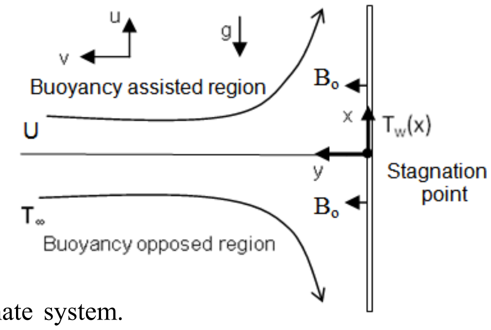

Fig. 1: Physical model and coordinate system.

Under these assumptions, along with the Boussinesq approximation, the boundary layer equations which govern the flow is given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\nu\frac{\partial^2 u}{\partial y^2}+k_0\left(\begin{array}{l} u\dfrac{\partial^3 u}{\partial x\partial y^2}+\dfrac{\partial u}{\partial x}\dfrac{\partial^2 u}{\partial y^2} \\ +\dfrac{\partial u}{\partial y}\dfrac{\partial^2 v}{\partial y^2}+v\dfrac{\partial^3 u}{\partial y^3}\end{array}\right) \tag{2}$$
$$\pm g\beta\left(T-T_\infty\right)+\sigma\frac{B_0^{\ 2}}{\rho}\left(U-u\right),$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

subject to the boundary conditions

$$u=0,\quad v=0,\quad T=T_w(x)\quad \text{at y=0}$$

$$u\to U_\infty\left(x\right),\quad \frac{\partial u}{\partial y}\to 0,\quad T\to T_\infty \quad \text{as y}\to\infty \tag{4}$$

where u and v are the velocity components along the x-and y-axes, respectively, T, g, , b, a, ρ, σ and k_0 are respectively the fluid temperature, acceleration due to the gravity, kinematic viscosity, thermal expansion coefficient, thermal diffusivity, fluid density, electrical conductivity and viscoelastic parameter. For the "±" sign in Eq. (2), the "+" sign corresponds to the assisting flow while the "–" sign corresponds to the opposing flow. The continuity, momentum and energy equations can be transformed into the corresponding ordinary differential equations by introducing the following variables:

$$\eta=\left(a/\nu\right)^{1/2}y,\quad u(x,y)=axf'\left(\eta\right),$$
$$v\left(x,y\right)=-\left(a\nu\right)^{1/2}f\left(\eta\right),\quad \theta\left(\eta\right)=\frac{T-T_\infty}{T_w-T_\infty} \tag{5}$$

By substituting (5) into Eqs.(1)-(3), Eq. (1) is identically satisfied and Eqs.(2) and (3) are transformed into:

$$f''' + ff'' - f'^2 + 1 + K\left(2f'f''' - f''^2 - ff^{iv}\right) \pm \lambda\theta + M\left(1 - f'\right) = \tag{6}$$

$$\theta'' + \Pr\left(f\theta' - f'\theta\right) = 0 \tag{7}$$

subject to the boundary conditions (4) which become

$$\begin{aligned} &f(0) = 0, \quad f'(0) = 0, \quad \theta(0) = 1, \\ &f'(\infty) = 1, \quad f''(\infty) = 0, \quad \theta(\infty) = 0 \end{aligned} \tag{8}$$

Here, primes denote differentiation with respect to η, $\Pr = \nu/\alpha$ is the Prandtl number, $M = B_0^2\sigma/(\rho a)$ is the magnetic parameter and the buoyancy or mixed convection parameter, $\lambda(\geq 0)$ is defined as

$$\lambda = \frac{g\beta b}{a^2} = \frac{g\beta\left(T_w - T_\infty\right)x^2/\nu^3}{U_\infty^2 x^2/\nu^2} = \frac{Gr_x}{\mathrm{Re}_x^2} \tag{9}$$

where $Gr_x = g\beta(T_w - T_\infty)x^2/\nu^3$ and $\mathrm{Re}_x = U_\infty x/\nu$ are the local Grashof and Reynolds numbers, respectively, and $K(\geq 0)$ is the dimensionless viscoelastic parameter, given by

$$K = \frac{k_0 a}{\rho} \tag{10}$$

The physical quantities of interest in this problem are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f = \frac{\tau_w}{\rho U_\infty^2}, \quad Nu_x = \frac{xq_w}{\alpha\left(T_w - T_\infty\right)}, \tag{11}$$

where τ_w and q_w are the wall shear stress and the surface heat flux, respectively, which are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} + k_0\left(u\frac{\partial^2 u}{\partial x\partial y} + v\frac{\partial^2 u}{\partial y^2} - 2\frac{\partial u}{\partial y}\frac{\partial v}{\partial y}\right)_{y=0},$$

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{12}$$

Substituting variables (5) into (12), we obtain

$$C_f \,\mathrm{Re}_x^{1/2} = \left[f'' + K\left(3f'f'' - ff'''\right)\right]_{\eta=0} = f''(0),$$

$$Nu_x / \mathrm{Re}_x^{1/2} = -\theta'(0). \tag{13}$$

RESULTS AND DISCUSSION

The transformed boundary layer equations (6) and (7) subject to the boundary conditions (8) have been solved numerically using an implicit finite-difference scheme known as the Keller-box method in conjunction with the Newton's linearization technique as described in the book by Cebeci and Bradshaw [18]. To validate the present method, the obtained results were compared with those of Hayat et.al [13] and the agreement is found to be good. Therefore, we are much confident that the developed code used in this study is suitable to solve the present problem discussed in this paper.

Table 1 presents the numerical values of the skin friction coefficient *f''*(0) and the local Nusselt number -θ'(0), respectively, for various values of the viscoelastic parameter *K* when Pr=0.2 for the case of buoyancy assisting flow (λ>0) and buoyancy opposing flow (λ<0). From Table 1, it can be seen that the values of $C_f \,\mathrm{Re}_x^{1/2}$ decrease as the viscoelastic parameter *K* increases, regardless of whether the flow is assisting or opposing. Comparatively, the values of the skin friction coefficient and the local Nusselt number are not much different between the assisting and opposing flows. However, the opposing flow gives smaller values compared to the assisting flow. Table 2 shows the effect of magnetic parameter *M* in the stagnation point flow adjacent to a vertical surface in a viscoelastic fluid with viscoelastic parameter *K*=1 for both assisting and opposing flows, respectively, when Pr=0.2. In this case, the viscoelastic parameter *K*=1 is held fixed and the effect of the imposed magnetic field is investigated. Clearly seen from Table 2, increment of the magnetic parameter *M* will also increase the magnitude of the skin friction coefficient and the local Nusselt number.

The profiles of the flow are shown in Figs. 2-5 below. Figs. 2 and 4 show the velocity profiles for various values of the magnetic parameter *M* for Pr=0.7 when the flow is assisted (*λ*=1) and opposed (*λ*=-1), respectively. For a particular value of *M*, the velocity boundary layer thickness increases monotonically with *η*, and becomes unity at the outside of the boundary layer, which actually satisfies the boundary condition $f'(\infty) \to 1$. As *M* increases, the boundary layer thickness decreases and hence leads to the increase of the drag friction.

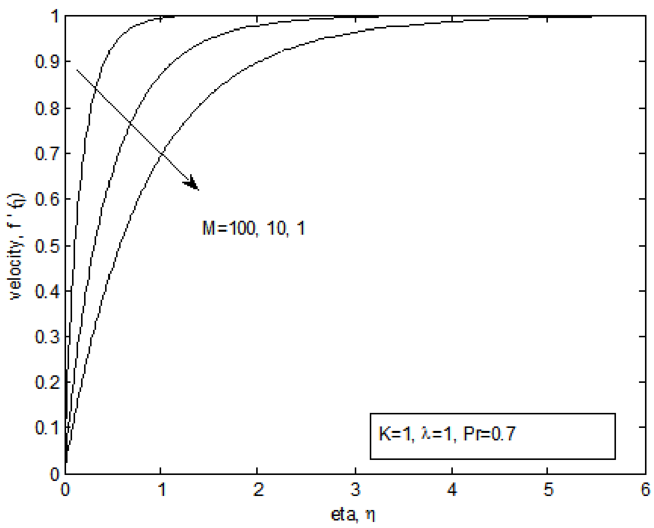

Fig. 2: Velocity profiles when Pr=0.7, *K*=1 and *λ*=1 (assisting flow) for various values of the magnetic parameter *M*.

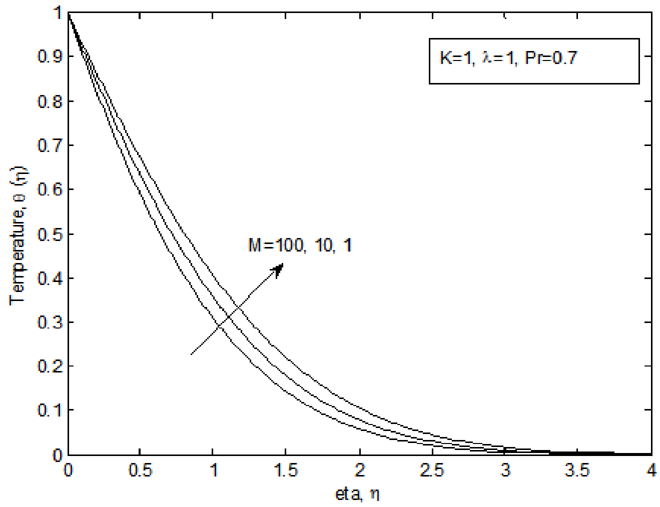

Fig. 3: Temperature profiles when Pr=0.7, K=1 and λ=1 (assisting flow) for various values of the magnetic parameter M.

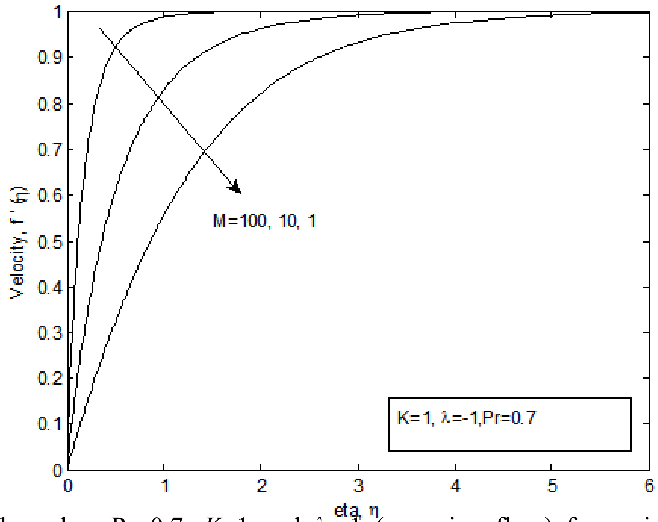

Fig. 4: Velocity profiles when Pr=0.7, K=1 and λ=-1 (opposing flow) for various values of the magnetic parameter M.

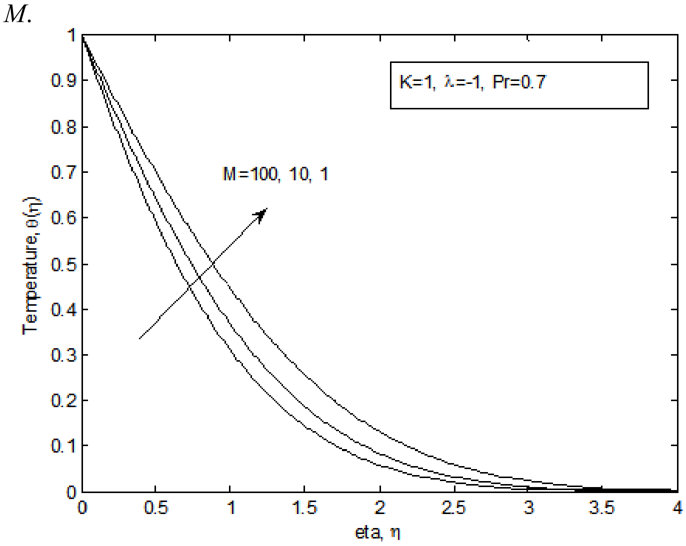

Fig. 5: Temperature profiles when Pr=0.7, K=1 and λ=-1 (opposing flow) for various values of the magnetic parameter M.

The temperature profiles when K=1 and Pr=0.7 for several values of M are shown in Figs. 3 and 5 for assisting and opposing flows, respectively. It is observed that the temperature and the thermal boundary layer thickness decreases as M increases, and hence leads to the increase of temperature gradient at the wall or local Nusselt number. Qualitative comparison shows that the profiles are not much different between the assisting and opposing flows. However, heating the plate will decrease the boundary layer formed and therefore increases the drag friction.

Conclusions:

A numerical study of the steady viscoelastic fluid towards stagnation point on a vertical surface with the effect of magnetic parameter M has been carried out. The governing partial differential equations were reduced to a set of ordinary differential equations by introducing the appropriate similarity variables and hence were solved for different values of the viscoelastic parameter and magnetic parameter. Both the buoyancy assisting flow (heated surface) and buoyancy opposing flow (cooled surface) situations were considered. It is found that the results for the skin friction coefficient and the local Nusselt number obtained for the assisting flow (heated plate) and opposing flow (cooled plate) cases are not much different and this led to the profiles forms for both cases to be alike.

Table 1: Values Of f^{2} (0) and -θ¢(0) Obtained When M=0, Pr =0.2, λ=0.2 (Assisting Flow) And λ=-0.2 (Opposing Flow) For various Values of the Viscoelastic Parameter K. Results in ( ) are Those of Hayat et al. [13].

K	Assisting flow (λ = 0.2)		Opposing flow (λ = -0.2)	
	$C_f Re_x^{1/2}$	$Nu_x / Re_x^{1/2}$	$C_f Re_x^{1/2}$	$Nu_x / Re_x^{1/2}$
0.5	0.9821 (0.9821)	0.4099 (0.4097)	0.8185 (0.8184)	0.3960 (0.3939)
1	0.8173 (0.8174)	0.3919 (0.3920)	0.6846 (0.6844)	0.3798 (0.3785)
2	0.6472 (0.6474)	0.3696 (0.3698)	0.5439 (0.5435)	0.3597 (0.3578)
3	0.5538	0.3551	0.4661	0.3470

Table 2: Values Of f^{2} (0) and -θ¢(0) Obtained When K=1, Pr =0.2, λ=0.2 (Assisting Flow) And λ=-0.2 (Opposing Flow) For various Values of MHD Parameter M.

M	Assisting flow (λ = 0.2)		Opposing flow (λ = -0.2)	
	$C_f Re_x^{1/2}$	$Nu_x / Re_x^{1/2}$	$C_f Re_x^{1/2}$	$Nu_x / Re_x^{1/2}$
1e+05	1.0512	0.4085	0.9469	0.3993
	2.2415	0.4585	2.1900	0.4555
	6.6431	0.5139	6.6246	0.5134

ACKNOWLEDGMENT

The authors gratefully acknowledge the financial support received from the Ministry of Higher Education, Malaysia (UKM-ST-07-FRGS0036-2009).

REFERENCES

Ariel, P.D., 2002. “On extra boundary condition in the stagnation point flow of a second-grade fluid,” Int. J. Eng. Sci., 40: 145-162.

Anwar, I., N. Amin and I. Pop, 2008. “Mixed convection boundary layer flow of a viscoelastic fluid over a horizontal circular cylinder,” Int. J. Non-Linear Mech., 43(9): 814-821.

Ayub, M., H. Zaman, M. Sajid and T. Hayat, 2008. “Analytical solution of stagnation-point flow of a viscoelastic fluid towards a stretching surface,” Commun. Nonlinear Sci. Numer. Simulation, 13(9): 1822-1835.

Beard, D.W. and K. Walters, 1964. “Elasto-viscous boundary layer flows,” Proc. Cambridge Philos. Society, 60: 667-674.

Beard, D.W. and K. Walters, 1964. “Elastco-viscous boundary layer flows. Part I: two dimension flow near a stagnation point,” Proc. Camb. Phil. Soc., 60: 667-674.

Cebeci, T. and P. Bradshaw, 1988. Physical and Computational Aspects of Convective Heat Transfer, Springer: New York.

Garg, V.K. and K.R. Rajagopal, 1990. “Stagnation point flow of a non-Newtonian fluid,” Mech. Res. Commun., 17: 415-421.

Hayat, T., Z. Abbas and I. Pop, 2008. “Mixed convection in the stagnation point flow adjacent to a vertical surface in a viscoelastic fluid,” Int. J. Heat Mass Transfer, 51(11-12): 3200-3206.

Ishak, A., R. Nazar and I. Pop, 2008. “ Magnetohydrodynamic (MHD) flow of a micropolar fluid towards a stagnation point on a vertical surface,” Computers and Mathematics with Application, 56: 3188-3194.

Li, D., F. Labropulu and I. Pop, 2009. “Oblique stagnation-point flow of a viscoelastic fluid with heat transfer,” Int. J. Non-linear Mech., 44(10): 1024-1030.

Mahapatra, T.R. and A.S. Gupta, 2004. “Stagnation-point flow of a viscoelastic fluid towards a stretching surface,” Int. J. Non-Linear Mech., 39(5): 811-820.

Mahapatra, T.R., S. Dholey and A.S. Gupta, 2007. “Momentum and heat transfer in the magnetohydrodynamic stagnation-point flow of a viscoelastic fluid toward a stretching surface,” Meccanica, 42: 263-272.

Oldroyd, J.G., 1950. “On the formulation of rheological equations of state,” Proc. R. Soc. London A, 200: 523-541.

Prasad, K.V., V. Dulal Pal, Umesh and N.S. Prasanna Rao, 2010. “The effect of variable viscosity on MHD viscoelastic fluid flow and heat transfer over a stretching sheet,” Commun. Nonlinear Sci. Numer Simulat., 15: 331-344.

Rajagopal, K.R., T.Y. Na and A.S Gupta, 1984. “Flow of a viscoelastic fluid over a stretching sheet,” Rheol. Acta, 23: 213-215.

Rajeswari, G. and S.L. Rathna, 1962. “Flow of a particular class of non-Newtonian viscoelastic and visco-inelastic fluids near a stagnation point,” *Z.* Angew. Math. Phys., 13: 43-57.

Ramachandran, N., T.S. Chen and B.F. Armaly, “Mixed convection in stagnation flows adjacent to vertical surface,” ASME J. Heat Mass Transfer, 110: 373-377.

Srivatsava, A.C., 1958. “The flow of a non-Newtonian liquid near a stagnation point,” Z. Angew. Math. Phys., 9: 80-84.