Australian Journal of Basic and Applied Sciences, 5(3): 337-345, 2011
ISSN 1991-8178

Differential Transform Method to Investigate Mass Transfer Phenomenon to a Falling Liquid Film System

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Abstract: In this paper, the application of the Differential Transform Method (DTM) in solving of the partial differential equation (PDE) governing mass transfer from a gas to a falling liquid film was investigated. A recursion in the transformed space was obtained by taking the differential transform over the PDE and its relevant boundary conditions together with some minor mathematical treatments. Afterwards, the inverse transform was easily applied to achieve the final solution. It was also shown that the aforementioned solution was exactly the same as the analytical solution resulted from the separation of variables method.

Key words: Differential transform method, falling liquid film, separation of variables method, partial differential equation

INTRODUCTION

Partial Differential Equations (PDEs) emerge in many engineering and scientific mathematical modeling problems. A multitude of methods have been proposed for solution of such equations. However, in many cases it is too complex to find an exact or analytical solution. That is when the approximate methods are the only choice. Differential Transform Method is a semi-analytical numerical approach which yields to Taylor series solution of differential equations. DTM was initially proposed by Zhou in his study on electrical circuits (Zhou, J.K., 1986). It was then extended for two-dimensional space by Jang *et al*. (2001) and Ayaz (2003). In 2004, Kurnaz *et al*. (2005) generalized the conventional DTM to n-dimensional space in order to enable the method to tackle higher order PDEs (Kurnaz *et al*. 2005). To speak about the advantages and generic nature of the DTM, it is worthwhile to mention that the method can be applied to linear and nonlinear PDEs of n-th order not requiring discretization, linearization or perturbation (Ayaz, F., 2003). Some examples of applications of DTM in the literature are as follows: Yu and Chen managed to solve the third-order nonlinear Blasius equation by DTM (Yu, L.T. and C.K. Chen, 1998). Chen and Liu investigated the applicability of DTM to steady state nonlinear conduction heat transfer problems (Chen, C.L. and Y.C. Liu, 1998). Ravi Kanth and Aruna applied DTM to successfully solve linear and nonlinear Klein-Gordon equation (Ravi Kanth, A.S.V. and K. Aruna, 2009) and Schrödinger equations (Ravi Kanth, A.S.V. and K. Aruna, 2009). Chen and Ho (1996) as well as Abdel-Halim Hassan (2002) exploited DTM to tackle eigenvalue problems. Bert utilized DTM to simulate heat conduction in tapered fins (Bert, C.W., 2002). Biazar and Eslami proposed DTM for solving quadratic Riccati differential equation and came up with satisfactory results (Biazar, J. and M. Eslami, 2010). Kaya conducted free vibration analysis of a rotating Timoshenko beam by DTM (Kaya, M.O., 2006). Cansu and Özkan applied DTM to solve blow up solutions of some linear wave equation with mixed non-linear boundary conditions (Cansu, Ü and O. Özkan, 2010).

In the current study, an effort is made to propose DTM for solving the PDE governing mass transfer from a stagnant gas atmosphere to an adjacent falling liquid film. Such a problem is of industrial significance as it is encountered in wetted-wall column gas absorbers and strippers of chemical operation units (Green D.W. and R.H. Perry, 2008). A precisely identical solution to the discussed PDE was also found by the separation of variables method, assuring the correctness of the analytical solution achieved via DTM.

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Basics of DTM:

Two-dimensional differential transform of the function can be defined as (Kurnaz, A., 2005):

$$\bar{W}(k,h) = \frac{1}{k!\,h!}\left[\frac{\partial^{k+h} w(x,y)}{\partial x^k \partial y^h}\right]_{\substack{x=0\\y=0}} \quad (1)$$

And the inverse differential transform of is in turn defined as:

$$w(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\{\bar{W}(k,h)x^k y^h\} \quad (2)$$

Table 1 summarizes some fundamental and useful operations pertaining to two-dimensional differential transform.

Table 1- Some operations of two-dimensional differential transform

Original function	Transformed function
$w(x,y) = \alpha u(x,y) \pm \beta v(x,y)$	$\bar{W}(k,h) = \alpha\bar{U}(k,h) \pm \beta\bar{V}(k,h)$; α, β are constant
$w(x,y) = \frac{\partial^{r+s}u(x,y)}{\partial x^r \partial y^s}$	$\bar{W}(k,h) = (k+1)(k+2)\cdots(k+r)(h+1)(h+2)\cdots(h+s)\,\bar{U}(k+r,h+s)$
$w(x,y) = x^m y^n$	$\bar{W}(k,h) = \delta(k-m)\delta(h-n)$ where $\delta(k-m) = \begin{cases}1, & k=m\\0, & k\neq m\end{cases}$ and $\delta(h-n) = \begin{cases}1, & h=n\\0, & h\neq n\end{cases}$
$w(x,y) = u(x,y)v(x,y)$	$\bar{W}(k,h) = \sum_{r=0}^{k}\sum_{s=0}^{h}\{\bar{U}(r,h-s)\bar{V}(k-r,s)\}$
$w(x,y) = x^m \sin(ay+b)$	$\bar{W}(k,h) = \frac{a^h}{h!}\delta(k-m)\sin(\frac{h\pi}{2}+b)$
$w(x,y) = x^m \cos(ay+b)$	$\bar{W}(k,h) = \frac{a^h}{h!}\delta(k-m)\cos(\frac{h\pi}{2}+b)$
$w(x,y) = x^m e^{ay}$	$\bar{W}(k,h) = \frac{a^h}{h!}\delta(k-m)$

Proofs to the above-mentioned theorems are fully available in references (Ayaz, F., 2003; Jafari, H., 2010).

Some illustrative numerical examples of utilizing DTM and the operations thereof can be found in references (Ravi Kanth, A.S.V. and K. Aruna, 2009; Chang, S.H. and I.L. Chang, 2009).

Analysis:

Writing mass balance over an infinitesimal element of the liquid in Cartesian coordinates while assuming:

1- The system is in a steady state condition (i.e. no variation of properties with time).
2- No chemical reaction between the gas and liquid occurs.
3- Fluid flows only along with the y-axis direction.
4- Concentration variation of component A is negligible in z direction.
5- Mass diffusivity *D* is constant.
6- Diffusion of the gas in the y direction is negligible in comparison with its downward movement due to bulk flow.
7- The solid wall is non-diffusible.
8- Liquid velocity is held constant as u_{avg}.

leads to the governing PDE and its boundary conditions below:

$$\frac{\partial^2 c}{\partial x^2} = \frac{u_{avg}}{D}\frac{\partial c}{\partial y} \tag{3}$$

BC1: $c(0,y) = C_{Ai}$ (4.a)

BC2: $c(x,0) = C_{A0}$ (4.b)

BC3: $\frac{\partial c}{\partial x}(\delta, y) = 0$ (4.c)

By defining $C(x,y) = c(x,y) - C_{Ai}$, the x-direction boundary conditions become homogenized:

$$\frac{\partial^2 C}{\partial x^2} = \frac{u_{avg}}{D}\frac{\partial C}{\partial y} \tag{5}$$

BC1: $C(0,y) = 0$ (6.a)

BC2: $C(x,0) = C_{A0} - C_{Ai}$ (6.b)

BC3: $\frac{\partial C}{\partial x}(\delta, y) = 0$ (6.c)

Assigning $\alpha = \frac{u_{avg}}{D}$ while applying differential transform to the governing equation gives:

$$(k+1)(k+2)\bar{C}(k+2,h) = \alpha(h+1)\bar{C}(k,h+1) \tag{7}$$

And therefore a recursive relation can be derived as:

$$\bar{C}(k,h+1) = \frac{(k+1)(k+2)}{\alpha(h+1)}\bar{C}(k+2,h) \tag{8}$$

Writing sine Fourier series for $f(x) = C_{A0} - C_{Ai}$ over the domain $0 < x < \delta$, we have:

$$C(x,0) = \frac{4(C_{A0} - C_{Ai})}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \left\{\frac{1}{n}\sin\left(\frac{n\pi x}{\delta}\right)\right\} \;\; ; \;\; 0 < x < \delta \tag{9}$$

From Taylors expansion series of $\sin\left(\frac{n\pi x}{\delta}\right)$ it follows that:

$$C(x,0) = \frac{4(C_{A0} - C_{Ai})}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \left\{\frac{1}{n} \sum_{k=1,3,5,\ldots}^{\infty} \left\{\frac{(-1)^{\frac{k-1}{2}}}{k!}\left(\frac{n\pi x}{\delta}\right)^k\right\}\right\} \;\; ; \;\; 0 < x < \delta \tag{10}$$

Consequently:

$$C(x,0)=\sum_{k=1,3,5,\ldots}^{\infty}\left\{x^k\sum_{n=1,3,5,\ldots}^{\infty}\left\{\frac{4(C_{A0}-C_{Ai})}{\pi}\frac{1}{n}\frac{(-1)^{\frac{k-1}{2}}}{k!}\left(\frac{n\pi}{\delta}\right)^{k}\right\}\right\}\;;\quad 0<x<\delta \tag{11}$$

From the definition of inverse differential transform:

$$C(x,0)=\sum_{k=0}^{\infty}\{\bar{C}(k,0)x^k\} \tag{12}$$

Comparing eq.11 and eq.12, it is deduced that:

$$\bar{C}(k,0)=\begin{cases}\dfrac{4(C_{A0}-C_{Ai})}{\pi}\displaystyle\sum_{n=1,3,5,\ldots}^{\infty}\left\{\frac{1}{n}\frac{(-1)^{\frac{k-1}{2}}}{k!}\left(\frac{n\pi}{\delta}\right)^{k}\right\} & ;\quad k\in odd\\ 0 & ;\quad k\in even\end{cases} \tag{13}$$

Using the recursive relation resulted from the governing equation, it can be written that:

$$\bar{C}(k,1)=\frac{(k+1)(k+2)}{\alpha}\bar{C}(k+2,0) \tag{14}$$

Therefore:

$$\bar{C}(k,1)=\begin{cases}\dfrac{4(C_{A0}-C_{Ai})}{\alpha\,\pi}\displaystyle\sum_{n=1,3,5,\ldots}^{\infty}\left\{\frac{1}{n}\frac{(-1)^{\frac{k+1}{2}}}{k!}\left(\frac{n\pi}{\delta}\right)^{k+2}\right\} & ;\quad k\in odd\\ 0 & ;\quad k\in even\end{cases} \tag{15}$$

In a similar fashion:

$$\bar{C}(k,2)=\begin{cases}\dfrac{4(C_{A0}-C_{Ai})}{2\alpha^2\,\pi}\displaystyle\sum_{n=1,3,5,\ldots}^{\infty}\left\{\frac{1}{n}\frac{(-1)^{\frac{k+3}{2}}}{k!}\left(\frac{n\pi}{\delta}\right)^{k+4}\right\} & ;\quad k\in odd\\ 0 & ;\quad k\in even\end{cases} \tag{16}$$

$$\bar{C}(k,3)=\begin{cases}\dfrac{4(C_{A0}-C_{Ai})}{6\alpha^3\,\pi}\displaystyle\sum_{n=1,3,5,\ldots}^{\infty}\left\{\frac{1}{n}\frac{(-1)^{\frac{k+5}{2}}}{k!}\left(\frac{n\pi}{\delta}\right)^{k+6}\right\} & ;\quad k\in odd\\ 0 & ;\quad k\in even\end{cases} \tag{17}$$

$$\bar{C}(k,h)=\begin{cases}\dfrac{4(C_{A0}-C_{Ai})}{h!\;\alpha^h\,\pi}\displaystyle\sum_{n=1,3,5,\ldots}^{\infty}\left\{\frac{1}{n}\frac{(-1)^{\frac{k-1+2h}{2}}}{k!}\left(\frac{n\pi}{\delta}\right)^{k+2h}\right\} & ;\quad k\in odd\\ 0 & ;\quad k\in even\end{cases} \tag{18}$$

By recalling the definition of inverse differential transform (eq.2) it can be written that:

$$C(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\{\bar{C}(k,h)x^k y^h\} \tag{19}$$

The third boundary condition (eq.6.c) imposes:

$$\frac{\partial C}{\partial x}(\delta,y) = 0 = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\{\bar{C}(k,h)k\,\delta^{k-1}y^h\} \tag{20}$$

Taking k=1 reduces the double-summation to a single one:

$$\sum_{h=0}^{\infty}\{\bar{C}(1,h)y^h\} = 0 \tag{21}$$

Hence, $\bar{C}(1,h) = 0$

Using the recursive relation (eq.14), it is yielded that:

$$\bar{C}(3,h) = 0$$

$$\bar{C}(5,h) = 0 \tag{22}$$

$$\vdots$$

$$\bar{C}(k \in odd,h) = 0$$

Eq.18 and eq.22 necessitate that for any k and h, $\bar{C}(k,h) = 0$ and thus for any given x and y, $C(x,y) = 0$. At this moment one may assume that the Differential Transform Method cannot be used to solve this problem. However, the failure to reach to non-trivial results stems from the fact that we ignored the boundary conditions imposed to this problem while writing the Fourier series for $f(x) = C_{A0} - C_{Ai}$ in eq.9. Therefore, a generalized form for Fourier series of the function shall be defined as:

$$f(x) = C_{A0} - C_{Ai} = a_0 + \sum_{n=0}^{\infty}\{a_n sin(\sqrt{\lambda_n}x) + b_n cos(\sqrt{\lambda_n}x)\} \tag{23}$$

And since the problem includes both Dirichlet and Neumann boundary conditions, the λ_n is to be expressed as:

$$\lambda_n = \left[\frac{\left(n+\frac{1}{2}\right)\pi}{\delta}\right]^2 ; n = 0,1,2,\ldots,\infty \tag{24}$$

Rewriting sine Fourier series for $f(x) = C_{A0} - C_{Ai}$ over, $0 < x < \delta$ we have:

$$C(x,0) = \frac{4(C_{A0} - C_{Ai})}{\pi}\sum_{n=0}^{\infty}\left\{\frac{1}{2n+1}\sin\left(\frac{(2n+1)\pi x}{2\delta}\right)\right\} \;;\; 0 < x < \delta \tag{25}$$

Repeating all the previous tasks (like what performed for eq.10 to eq.18), it is yielded that:

$$\bar{C}(k,h) = \begin{cases} \dfrac{4(C_{A0}-C_{Ai})}{h!\ \alpha^h\ \pi}\displaystyle\sum_{n=0}^{\infty}\left\{\frac{1}{2n+1}\frac{(-1)^{\frac{k-1+2h}{2}}}{k!}\left(\frac{(2n+1)\pi x}{2\delta}\right)^{k+2h}\right\} & ;\quad k \in odd \\ 0 & ;\quad k \in even \end{cases} \tag{26}$$

Applying the inverse differential transform, we have:

$$C(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\begin{cases} \dfrac{4(C_{A0}-C_{Ai})\ x^k y^h}{h!\ \alpha^h\ \pi}\displaystyle\sum_{n=0}^{\infty}\left\{\frac{1}{2n+1}\frac{(-1)^{\frac{k-1+2h}{2}}}{k!}\left(\frac{(2n+1)\pi x}{2\delta}\right)^{k+2h}\right\} & ;\quad k \in odd \\ 0 & ;\quad k \in even \end{cases} \tag{27}$$

$$C(x,y) = \sum_{k=1,3,5,\dots}^{\infty}\ \sum_{h=0}^{\infty}\left\{\frac{4(C_{A0}-C_{Ai})\ x^k y^h}{h!\ \alpha^h\ \pi}\sum_{n=0}^{\infty}\left[\frac{1}{2n+1}\frac{(-1)^{\frac{k-1+2h}{2}}}{k!}\left(\frac{(2n+1)\pi x}{2\delta}\right)^{k+2h}\right]\right\} \tag{28}$$

Rearranging the summations, we obtain:

$$C(x,y) = \frac{4(C_{A0}-C_{Ai})}{\pi}\sum_{n=0}^{\infty}\left[\frac{1}{n}\sum_{k=1,3,5,\dots}^{\infty}\left\{\frac{(-1)^{\frac{k-1}{2}}}{k!}\left(\frac{(2n+1)\pi x}{2\delta}\right)^{k}\right\}\sum_{h=0}^{\infty}\left\{\frac{\left(-\frac{\left(\frac{(2n+1)\pi}{2\delta}\right)^2}{\alpha}y\right)^h}{h!}\right\}\right] \tag{29}$$

Recalling Talyor expansion series of sine and exponential functions, it is revealed that:

$$C(x,y) = \frac{4(C_{A0}-C_{Ai})}{\pi}\sum_{n=0}^{\infty}\left[\frac{1}{2n+1}\sin\left(\frac{(2n+1)\pi}{2\delta}x\right)exp\left(-\frac{1}{\alpha}\left(\frac{(2n+1)\pi}{2\delta}\right)^2 y\right)\right] \tag{30}$$

Finally:

$$c(x,y) = C_{Ai} + \frac{4(C_{A0}-C_{Ai})}{\pi}\sum_{n=0}^{\infty}\left\{\frac{1}{2n+1}\sin\left(\frac{(2n+1)\pi}{2\delta}x\right)exp\left[-\frac{D}{u_{avg}}\left(\frac{(2n+1)\pi}{2\delta}\right)^2 y\right]\right\} \tag{31}$$

for any x in $0 \le x \le \delta$.

As it is observed, the solution by the Differential Transform Method is now exactly identical to that of by the Separation of Variables Method (see Appendix A).

Conclusion:

In the current study, two-dimensional Differential Transform Method was used to solve a PDE stemmed from mathematical modeling of mass transfer to falling liquid films. The solution procedure entailed three steps, namely, transformation of the differential equation and its boundary conditions (which converts them into algebraic equations), finding a recurrence relation in the transformed space, and finally utilization of the inverse differential transform. It was shown that the DTM had led to the same analytical solution obtained from the separation of variables method. DTM offers benefits in terms of variety and ease of applicability. Another valuable point in DTM is that unlike the case with integral transforms such as Fourier or Laplace transform, the corresponding inverse transform is highly simple to compute and extremely fast.

Appendix A: Derivation of the solution via Separation of Variables Method

$$\frac{\partial^2 C}{\partial x^2} = \frac{u_{avg}}{D}\frac{\partial C}{\partial y} \qquad (A.1)$$

BC1: $C(0,y) = 0$ (A.1a)

BC2: $C(x,0) = C_{A0} - C_{Ai}$ (A.1b)

BC3: $\frac{\partial C}{\partial x}(\delta, y) = 0$ (A.1c)

According to the Product Rule, the solution to (A.1) can be written as:

$$C(x,y) = X(x)Y(y) \qquad (A.2)$$

Substituting (A.2) into (A.1) we obtain:

$$\frac{\frac{d^2X}{dx^2}}{X} = \frac{u_{avg}}{D}\frac{\frac{dY}{dy}}{Y} = -\lambda \qquad (A.3)$$

from (A.3) it is followed that:

$$Y(y) = k\exp(-\frac{D}{u_{avg}}\lambda y) \qquad (A.4)$$

and

$$X(x) = a\sin(\sqrt{\lambda}x) + b\cos(\sqrt{\lambda}x) \qquad (A.5)$$

(A.1a) necessitates that b=0 in (A.5). Also, from (A.1c) it is found that:

$$\cos(\sqrt{\lambda}x) = 0 \rightarrow \sqrt{\lambda_i} = \frac{2i+1}{2\delta}\pi, \quad i = 0,1,2,\ldots \qquad (A.6)$$

Thus, (A.2) can be rewritten as:

$$C(x,y) = \sum_{i=0}^{\infty} \left\{ A_i \sin\left(\frac{2i+1}{2\delta}\pi x\right) \exp\left[-\frac{D}{u_{avg}}\left(\frac{2i+1}{2\delta}\pi\right)^2 y\right]\right\} \tag{A.7}$$

By using (A1.b) we obtain:

$$A_i = \frac{(C_{A0} - C_{Ai}) \int_0^{\delta} \sin\left(\frac{2i+1}{2\delta}\pi x\right) \mathrm{dx}}{\int_0^{\delta} \sin^2\left(\frac{2i+1}{2\delta}\pi x\right) \mathrm{dx}}, \quad i = 0,1,2, \ldots \tag{A.8}$$

which can be reduced to:

$$A_i = \frac{4(C_{A0} - C_{Ai})}{(2i+1)\pi} \tag{A.9}$$

Hence, the analytical solution is obtained as:

$$C(x,y) = \frac{4(C_{A0} - C_{Ai})}{\pi} \sum_{i=0}^{\infty} \left\{ \frac{1}{2i+1} \sin\left(\frac{(2i+1)\pi}{2\delta} x\right) exp\left[-\frac{D}{u_{avg}}\left(\frac{2i+1}{2\delta}\pi\right)^2 y\right]\right\}; \; 0 \le x \le \delta \tag{A.10}$$

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