Australian Journal of Basic and Applied Sciences, 5(3): 309-316, 2011
ISSN 1991-8178

Simple Method for Analysis of Tube Frame by Consideration of Negative Shear Lag

[1]Reza Mahjoub, [2]Reza Rahgozar, [2]Hamed Saffari

[1]Academic Member, Faculty of Engineering, Islamic Azad University, Khoramabad Branch, Khoramabad, Iran.
[2]Departement of Civil Engineering, ShahidBahonar University of Kerman, Kerman, Iran

Abstract: Tubular frame structure is one of the most efficient systems in tall buildings underlateral load. The analysis of these structures usually involves considerable time andeffort due to large number of members and joints. A method for evaluating shear lagand estimating stress of frame elements is presented. In this method,tube frame is assumed as a web and flange panel and then by consideringdeformation functions for web and flange frames and writing their stress relations aswell as use of minimum energy basis, functions are presented for lateral and verticaldisplacement of the structure. Relations suggested in this paper are capable ofconsidering shear lag both in flange and web frames in the height of frame. The simplicity and accuracy of the proposed method is demonstrated through the numerical analysis of several structures. Furthermore, coefficients are given in terms of height to consider shear lag variation.

Key words: Tubular frame, Tall building, Lateral load, Shear lag, Simple analysis.

INTRODUCTION

Frame tube structures are widely accepted as an economical system for high-risebuildings (Coull, A. and N.K. Subedi, 1971; Foutch, D.A. and P.C. Chang, 1982). In its basic form, the system consists of closely spaced exterior columns along the periphery interconnected by deep spandrel beams of each floor level. This produces a system of rigidly jointed orthogonal frame panels forming a rectangular tube which acts as a cantilever hollow box. Frame tube acts like a hollow boxed beam under lateral loads such as wind and earthquake. The occurance of shear lag has long been recognized in hollow box girder as well as in tubular buildings. The most existing exact method of analyzing (3-D software) is very expensive due to modeling and the large number of degree of freedom. In tubular buildings, flexibility of spandrel beams produces shear lag phenomenon with the effect of increasing the axial stresses in the corner columns and of reducing axial stress in the columns toward the center of flange panel of the orthogonal frame in the bottom of structure (Fig.1). Therefore, approximated methods are suggested which are useful in primary design and stress estimation of the structure. Different methods of simulation, which consider elastic behaviors of perimeter frames as equivalent membranes, are presented (Chan, P.C.K., 1974; Coull, A. and A.A. Ahmed, 1978; Coull, A. and B. Bose, 1975; Coull, A. and B. Bose, 1976; Ha, K.H., 1978; Kang-Kun, L., 2001). An anomaly in the shear lag behavior of a cantilever box girder has been observed (Foutch, D.A. and P.C. Chang, 1982). In the region beyond about one-quarter the cantilever length from the built in end, the bending stress near the web is smaller than that near the center of the flange. This phenomenon is opposite to the positive shear lag and is called negative shear lag. The behavior of frame-tube is complicated since the positive and negative shear lag phenomenon occurred respectively at the bottom and the top of structure (Kwan, A.K.H., 1994). Recently, some researches have done to propose methods to evaluate the variation of shear lag in tall building (Lee, K.K., 2000; Rahgozar, R., 2010; Rahgozar, R., Y. Sharifi, 2009; Kaviani, P., 2008). Deformations which are related to shear lag may have improper influence on unloaded members of the structure. Torsion in panel of each story may intensify existing deformations and stresses. This intensification threatens safety and stability of the structure. In this paper by considering separate deformation functions for web and flange frames and then writing stress-deformations relations as well as the use of minimum energy basis, functions are suggested for lateral and vertical displacements.

Corresponding Author: Reza Mahjoub, Academic Member, Faculty of Engineering, Islamic Azad University, Khoramabad Branch, Khoramabad, Iran.

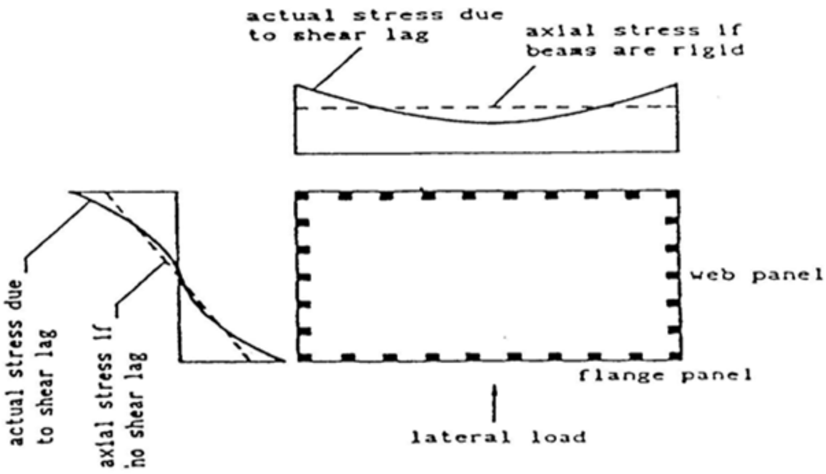

Fig. 1: Distribution of axial stress in framed tube structure.

Modeling Method:

Modeling method for frame panels is carried out as orthotropic equivalent members in a way that perimeter frame could be analyzed as a continuous structure. Perimeter frame structure shown in Figure 2 can be considered as two web panels which are parallel to lateral loads direction and two flange frames which are orthogonal to that in accordance with the following assumptions: (1) with regards to the stiffness of floors, out of plane behaviors are negligible in comparison with in-plane behaviors of frames; (2) beams and columns dimensions are similar, so that frame panel can be modeled as continuous equivalent membranes (Kwan, A.K.H., 1994).

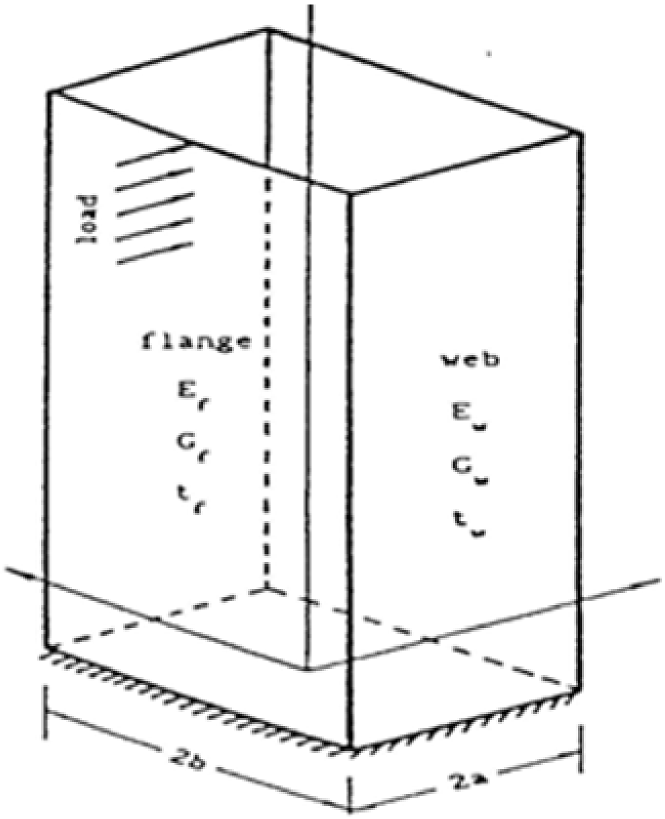

Fig. 2: Typical orthotropic panels of framed tube structure

Arithmetic Relations:

Stress distributions may not be linear in the members due to the shear lag occursin web and flange panels. The axial stress distribution of web and flange in this studyare considered respectively as a quadratic and cubic functions. Therefore, the intensityof axial stresses in web depend on the intensity of axial load in flange. Axialdeformations in web (*w*) and flange (*w*') can be represented by the following equations:

$$w = \emptyset\alpha\left[\left(1-\frac{\alpha}{\lambda}\right)\frac{x}{a}+\frac{\alpha}{\lambda}\left(\frac{x}{a}\right)^3\right] \quad (1)$$

$$w = \emptyset\alpha\left[\left(1-\frac{\alpha}{\lambda}\right)\frac{x}{a}+\frac{\alpha}{\lambda}\left(\frac{x}{a}\right)^3\right] \quad (2)$$

Where φ is the rotation of plane section which connects four sides of tubular structure which were originally in a horizontal plane. α and β are dimensionless coefficients of shear lag which show degrees of shear lag in web and flange planes. λ is the coefficient of shear lag variation in height. Relations for section rotation (φ), axial and shear strains in web and flange planes are given by the following equationsrespectively:

$$\phi = \frac{1}{EI}\int_0^z M\,dz \quad (3)$$

$$\varepsilon_z = \frac{\partial w}{\partial z} \qquad \varepsilon_z = \frac{\partial w}{\partial z} \quad (4)$$

$$\varepsilon_z = \frac{\partial w}{\partial z} \qquad \varepsilon_z = \frac{\partial w}{\partial z} \quad (5)$$

The strain energy of perimetr frame is calculated as follows:

$$\Pi_e = \int_0^h\int_{-a}^{a} t_w(E\varepsilon_z^2 + G\gamma_{xz}^2)\,dx\,dz + \int_0^h\int_{-b}^{b} t_f\left(E\varepsilon_z'^2 + G\gamma_{yz}^2\right)dy\,dz \quad (6)$$

The potential energy of the applied lateral load is given by the following equations for different load cases:
Case 1: Single load P at the top of the structure:

$$\Pi_p = Pu(h) \quad (7)$$

Case 2: Uniformly distributed load with P defined as the intensity of load per unit height:

$$\Pi_p = -\int_0^h Pu(z)\,dz \quad (8)$$

Case 3: Linearly distributed load (triangular) with T defined as the intensity of load per unit height at top and zero intensity at the bottom:

$$\Pi_p = \int_0^h T\frac{z}{h}u(z)dz \quad (9)$$

Coefficients α and β in Equations (1) and (2) are assigned with the use of minimum energy basis and λ is calculated with the help of numerical study of several structures.

$$\alpha = \alpha_1\left(1-\frac{z}{H}\right)^2 + \alpha_2\left[2\frac{z}{H}-\left(\frac{z}{H}\right)^2\right] \tag{10}$$

$$\alpha = \alpha_1\left(1-\frac{z}{H}\right)^2 + \alpha_2\left[2\frac{z}{H}-\left(\frac{z}{H}\right)^2\right] \tag{11}$$

The values of „andare given in Table 1:

Table 1: Shear lag coefficients and (Kwan, A.K.H., 1994).

Type of load	α	β
Single load at the top of the structure	$\alpha_1 = \frac{1.17m_w + 1.00}{m_w^2 + 2.67m_w + 0.57}$ $\alpha_2 = \frac{0.29m_w + 1.00}{m_w^2 + 2.67m_w + 0.57}$	$\beta_1 = \frac{3.5m_f + 12.60}{m_f^2 + 11.20m_f + 10.08}$ $\beta_2 = \frac{0.88m_f + 12.60}{m_f^2 + 11.20m_f + 10.08}$
Uniform load	$\alpha_1 = \frac{2.57m_w + 1.12}{m_w^2 + 2.94m_w + 0.64}$ $\alpha_2 = \frac{0.03m_w + 1.12}{m_w^2 + 2.94m_w + 0.64}$	$\beta_1 = \frac{7.72m_f + 14.15}{m_f^2 + 12.35m_f + 11.32}$ $\beta_2 = \frac{0.08m_f + 14.15}{m_f^2 + 12.35m_f + 11.32}$
Linearly distributed load	$\alpha_1 = \frac{2.22m_w + 1.09}{m_w^2 + 2.86m_w + 0.62}$ $\alpha_2 = \frac{0.10m_w + 1.09}{m_w^2 + 2.86m_w + 0.62}$	$\beta_1 = \frac{6.67m_f + 13.71}{m_f^2 + 12.01m_f + 10.97}$ $\beta_2 = \frac{0.29m_f + 13.71}{m_f^2 + 12.01m_f + 10.97}$

$$m_w = \frac{G_w H^2}{E_w a^2} \qquad m_f = \frac{G_f H^2}{E_f b^2} \tag{12}$$

Where G_w and G_f are equivalent shear Modulus of web and flange panels, E_w and E_f are equivalent Young's Modulus of web and flange respectively From the preceding relationships it can be concluded that the shear lag coefficients in each panels depends on elastic properties of materials and the height of structure. It can be observed that shear lag coefficients increase as the dimensions of the structure (2a, 2b, H) increase.

4- Calculating the Variation of Shear Lag in Height (λ):

Shear lag phenomenon is observed not only in tube frames but also in cantilever boxed beams (Foutch, D.A. and P.C. Chang, 1982). Numerical method is used to calculate the variations of shear lag in height. The values of α and β do not depend on λ. Stress values can be derived by applying the Hook's law:

$$\sigma_{web} = E\frac{\partial_w}{\partial_z} \qquad \sigma_{flange} = E\frac{\partial_{w'}}{\partial_z} \tag{13}$$

The unknowns values of α and β are derived from relations (10), (11) and Table 1 which leads to lengthy relations for stresses. Each of these equations for different type of loading has four unknown parameters known as: z (height variable), x or y, longitudinal variables respectively for web and flange and λ, shear lag coefficient in height. For example, results obtained from analysis of 5 structures subjected to uniform lateral load are shown in Figures 3 and 4.

In order to calculate λ several structures should be analyzed for different types of loading and exact value of stresses should be evaluated in different levels of the structure. Each of the equations related to the stresses at point (x, z) for web and (y, z) for flange can be solved by having the results of exact analysis and consequently λ is found. The values of λ are calculated through equalizing the values of stresses obtained from Equation (12) with the exact values determind from the result of exact analysis of the structure. Functions are exposed both for web and flange frame after organizing the values of λ according to z/h and nonlinear regression.

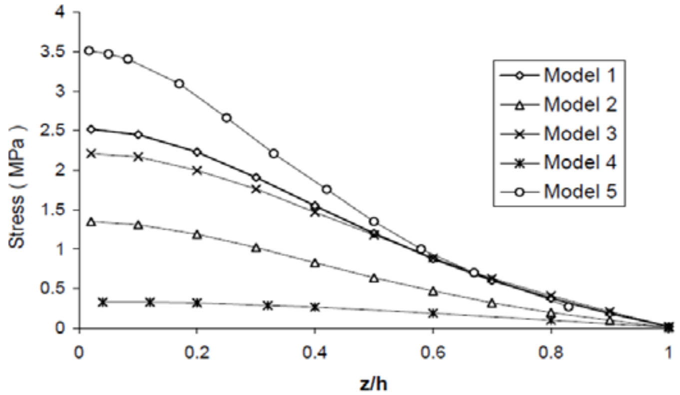

Fig. 3: Stress in middle column of flange panel at the height

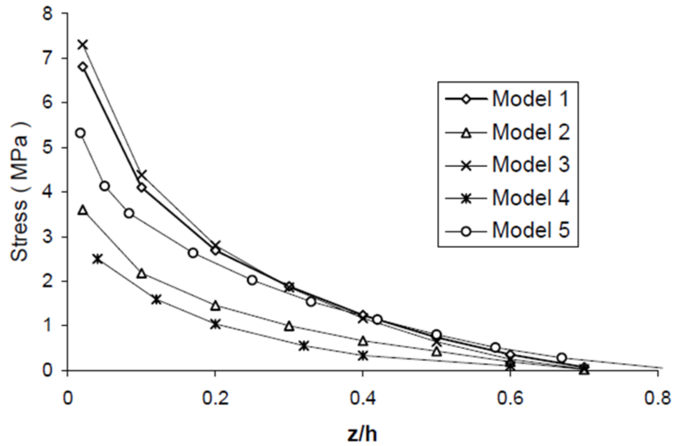

Fig. 4: Stress in corner column of web panel at the height

Where *X=z/h* in Table 2. In web frame equations, λ belongs to the web end point and the point of web and flange intersection and stress in web middle point is zero. In flange frame equations λ's are related to middle point of flange frame. Stress in flange frame endpoint is the same as what is resulted from web equations. Therefore according to the calculated stresses in these points axial stress diagrams of columns in perimeter frames can be easily plotted. According to Equations (13), it can be shown:

$$\sigma_{web} = Ea\frac{d\emptyset}{dz}\left[\left(1-\frac{\alpha}{\lambda}\right)\frac{x}{a}+\frac{\alpha}{\lambda}\left(\frac{x}{a}\right)^3\right]\sigma_{flange} = Ea\frac{d\emptyset}{dz}\left[\left(1-\frac{\beta}{\lambda}\right)+\frac{\beta}{\lambda}\left(\frac{y}{b}\right)^2\right] \quad (14)$$

The values of are given by Equations (15), (16) and (17) for different typesof loading:

$$\frac{d\phi}{dz} = \frac{M}{EI} = \frac{p(h-z)}{EI} \quad (15)$$

Table 2: The functions of λ in web and flange panel

Type of Load	λ
Single load at the top of the structure	$\lambda_{web} = \dfrac{1}{a + bX^2 + \dfrac{c}{X}}$ $\lambda_{flange} = e^{(a+cX)/(1+bX)}$
Uniform load	$\lambda_{web} = a + \dfrac{b}{X} + \dfrac{c}{X^2}$ $\lambda_{flange} = \dfrac{1}{a + bX^2}$
Linearly distributed load	$\lambda_{web} = a + bX^3 + \dfrac{c}{\sqrt{X}}$ $\lambda_{flange} = a + bX + cX^2 + dX^3 + eX^4 + fX^5$

Table 3: Coefficient of λ 's function

Types of loading	a		b		c		d		e		f	
	Web	Flange	Web	Flange	Web	Flange	Web	Flange	Web	Flange	Web	Flange
Single load	0.023	0.4	0.25	-0.98	0.0005	0.346	-	-	-	-	-	-
Uniform load	4176	1.09	2326	-4.6	212.42	-	-	-	-	-	-	-
Linearly load	0.012	7.41	0.11	7.42	-0.047	76.32	-	244.3	-	507.3	-	359.5

$$\frac{d\phi}{dz} = \frac{M}{EI} = \frac{p(h-z)^2}{2EI} \tag{16}$$

$$\frac{d\phi}{dz} = \frac{M}{EI} = \frac{T(h-z)^2}{6h}\left(\frac{z+2h}{EI}\right) \tag{17}$$

Equation (18) is defined by using the equilibrium of bending moments and axial loads in web and flange panels in order to calculate the unknown equivalent amount of EI.

$$EI = \frac{4}{3}Et_w a^3\left(1 - \frac{2}{5}\alpha\right) + 4Et_f a^2 b\left(1 - \frac{2}{3}\beta\right) \tag{18}$$

5- Numerical Study:

To illustrate the application of the proposed method, it is compared with exact and Kwan's methods by using a 40 story reinforced concrete building with the following specifications: Height of stories 3 meters, column spacing 2.4 meter, dimensions of all beams and columns 0.8 x 0.8 meter, uniformly distributed load 120 KN/m, E=20 GPa, equivalent value of G=1.441 GPa, 2a=30 m and 2b=35 m. According to the Figure 5 it is observed that suggested relations are perfectly capable of taking the negative shear lag phenomenon into account at the top of the structure. It is obvious in Figure 6 that suggested relations have accuracy in height of structure and by concerning Kwan's method the results are more similar to the exact method.

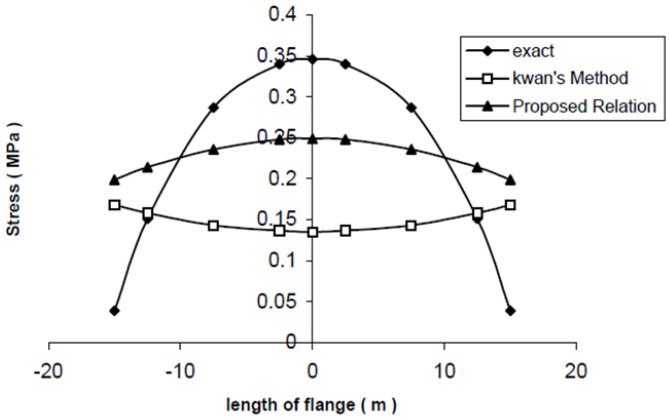

Fig. 5: Stress in middle column of flange panel in height of 90 m.

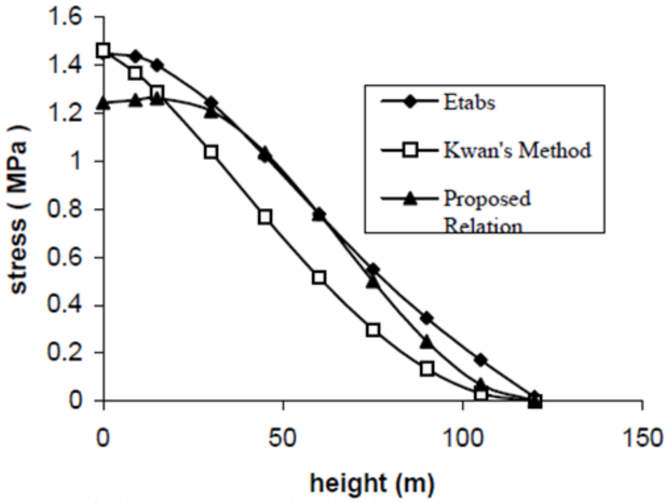

Fig. 6: Stress in middle column of flange panel at the height of structure

6- Conclusion:

The presented method in this paper is capable of explaining stress distribution with high accuracy. The following results can be obtained:

1. According to the proposed relations, stress in each column can be calculated by theircoordinates.
2. Proposed equations are capable considering positive shear lag at the bottom of the structure with high accuracy for both web and flange frames. The percentage of error is about 8–15. By considering different models in height of the structure, it is revealed that these equations can consider negative shear lag at top of the structure and has more accuracy than those of other references.
3. By plotting α and β diagrams in terms of the beams moment of inertia, it is observed that by increasing beams moment of inertia, positive shear lag effects decrease but low slope of the curve shows that it is not an optimized solution by itself to decrease shear lag effects.
4. By drawing α and β diagrams in terms of the columns' section, it is observed that by increasing column's section, shear lag effects reduced noticeably in system and the curve slope shows that section increasing of columns has better influence on shear lag than increasing of beams moment of inertia.
5. By plotting stress diagrams in flange frame for different models at the height of the structure and by considering the influences of parameters variation on the inflection point of shear lag, it is observed that changing columns' dimension play the most effective role and it is located at the (0.25 - 0.33)H.

REFERENCES

Chan, P.C.K., W.K. Tso and A.C. Heidebrecht, 1974. Effect of Normal Frames on Shear Walls. Building Sci., 9: 197-209.

Coull, A. and A.A. Ahmed, 1978. Deflections of Frame-Tube Structures. J. Struct.Div., ASCE, 104(5): 857-862.

Coull, A. and B. Bose, 1975. Simplified Analysis of Frame – Tube Structures. J. Struct.Div., ASCE, 101(11): 2223-2240.

Coull, A. and B. Bose, 1976. Torsion of Frame–Tube Structures. J. Struct. Div., ASCE, 102(12): 2366-2370.

Coull, A. and N.K. Subedi, 1971. Framed-Tube Structures for High-Rise Buildings. J.Struct. Div., ASCE, 104(9): 1495-1505.

Foutch, D.A. and P.C. Chang, 1982. A Shear Lag Anomaly. J. Struct. Eng., 108(7): 1653-1658.

Ha, K.H., P. Fazio and O. Moselhi, 1978. Orthotropic Membrane for Tall Building Analysis. J. Struct. Div., ASCE, 104(9): 1495-1505.

Kaviani, P., R. Rahgozar, H. Saffari, 2008. Approximate Analysis of TallBuildings UsingSandwich Beam Models with Variable Cross-Section. Struct. Design Tall Spec. Build., 17: 401-418.

Kwan, A.K.H., 1994. Simple Method for Approximate Analysis of Framed Tube Structures. J. Struct. Eng., ASCE, 120(4): 1221-1239.

Kang-Kun, L., L. Yew-Chaye and G. Hong, 2001. Simple Analysis of Framed-TubeStructures with Multiple Internal Tubes. J. Struct. Eng., ASCE, 127(4): 450-460.

Lee, K.K., H. Guan, Y.C. Loo, 2000. Simplified Analysis of Shear-lag in Framed-Tube Structures with Multiple Internal Tubes. Computational mechanics, 26: 447-458.

Rahgozar, R., A.R. Ahmadi, Y. Sharifi, 2010. A Simple Mathematical Model for Approximate Analysis of Tall Buildings.Applied Mathematical Modeling, 34: 2437-2451.

Rahgozar, R., Y. Sharifi, 2009. An Approximate Analysis of Combined System of Framed Tube, Shear Core and Belt Truss in High rise Buildings. Struct. Design Tall Spec. Build., 18: 607-624.