Australian Journal of Basic and Applied Sciences, 5(3): 281-294, 2011
ISSN 1991-8178

Design Adaptive Sliding Mode Controller by Type-2 Fuzzy Modeling

[1]M. Mohammad Abadi, [2]H. Chahkandi Nejad, [3]R. Jahani, [1]A. Zare

[1]Electrical Engineering Department Islamic Azad University, Gonabad Branch, Gonabad, Iran.
[2]Electrical Engineering Department Islamic Azad University, Birjand Branch, Birjand, Iran.
[3]Electrical Engineering Department Islamic Azad University, South Tehran Branch, Tehran, Iran.

Abstract: In this paper, an adaptive type-2 fuzzy sliding mode control algorithm is proposed for a class of continuous time unknown nonlinear systems. the proposed adaptive type -2 fuzzy logic controller takes advantages of every three method sliding mode control, fuzzy control and proportional integral (PI) control. The stability analysis for the proposed control algorithm is provided. An inverted pendulum with variation of pole characteristics, and are adopted to illustrate the validity of the proposed method. The simulation results show that the adaptive type-2 fuzzy sliding mode controller achieves the best tracking performance in comparison with the adaptive type-1 fuzzy sliding mode controller.

Key words: Adaptive type-2 fuzzy sliding mode control algorithm; Robust control;

INTRODUCTION

Fuzzy logic control is a technique of incorporating expert knowledge in designing a controller. Past research of universal approximation theorem (L.X. Wang. 1993) shown that any nonlinear function over a compact set with arbitrary accuracy can be approximated by a fuzzy system.

It is well known that the sliding mode control method provides a robust controller for nonlinear dynamic systems (J.E. Slotine, *et al*. 1991). However, it inherits a discontinuous control action and hence chattering phenomena will take place when the system operates near the sliding surface. One of the common solutions for eliminating this chattering effect is to introduce a boundary layer neighboring the sliding surface (V.I. Utkin. 1977). This method can lead to stable closed loop system without the chattering problem, but there exists a finite steady state error due to the finite steady state gain of the control algorithm..

The adaptive fuzzy controller incorporating the fuzzy logic and the sliding mode control (SMC) (V.I. Utkin. 1977) for ensuring stability and consistent performance is an active research topic of the fuzzy control. One of the advantages of this control strategy is insensitive to modeling uncertainty and external disturbances. Many adaptive fuzzy sliding mode control (AFSMC) schemes have been proposed and the chattering phenomena in the controlled system can be avoided by using the fuzzy sliding surface in the reaching condition of the SMC (S.W.Kim, *et al*. 1995). However, these features make the number of fuzzy rules increasing with the complexity of the fuzzy sliding surface involved. As the sliding mode control law can separated into two parts i.e. the equivalent control and the switching control (V.I. Utkin. 1977). The role of the controller is to schedule these two components under different operating conditions. In order to improve the steady state performance of the AFSMC, an adaptive fuzzy logic controller combining a proportional plus integral (PI) controller and the SMC is considered in this paper. The proposed control scheme provides good transient and robust performance. Moreover, as the proposed controller integrates the PI control with the SMC, the chattering phenomenon can be avoided. In this paper, it is proved that the closed-loop system is globally stable in the Lyapunov sense and the system output can track the desired reference output asymptotically with modeling uncertainties and disturbances.

2 – Problem Statement:

Consider a general class of SISO n-th order nonlinear systems as follow form (L.X. Wang. 1993):

$$x^{(n)} = f(\underline{x}, t) + g(\underline{x}, t)u + d(t)$$

$$y = x \qquad (1)$$

Corresponding Author: M. Mohammad Abadi, Electrical Engineering Department Islamic Azad University, Gonabad Branch, Gonabad, Iran.

where f and g are unknown nonlinear functions, $\underline{x} = [x, \dot{x}, \ldots, x^{(n-1)}]^T = [x_1, x_2, \ldots, x_n]^T \in R^n$ is the state vector of the systems which is assumed to be available for measurement, $u \in R$ and $y \in R$ are the input and the output of the system, respectively, and d(t) is the unknown external disturbance. We have to make an assumption that have upper bound D, that is, d(t)≤D. We require the system (1), to be controllable, the input gain $g(\underline{x}, t) \neq 0$ is necessary. Hence, without loss of generality, we are assumed $(\underline{x}, t) > 0$. The control problem is to obtain the state x for tracking a desired state x_d in the presence of model uncertainties and external disturbance with the tracking error

$$\underline{e} = \underline{x} - \underline{x}_d = [e, \dot{e}, \ldots, e^{(n-1)}]^T \in R^n \tag{2}$$

Define a sliding surface in the space of the error state as

$$s(\underline{e}) = c_1 e + c_2 \dot{e} + \cdots + c_{n-1} e^{(n-2)} + e^{(n-1)} = \underline{c}^T \underline{e} \tag{3}$$

where $\underline{c} = [c_1, c_2, \ldots, c_{n-1}, 1]^T$ are the coefficients of the Hurwitiz polynominal $h(\lambda) = \lambda^{n-1} + c_{n-1}\lambda^{n-2} + \cdots + c_1$, i.e. all the roots are in the open left half-plane and λ is a Laplace operator. If the initial condition e(0)=0, the tracking problem $\underline{x} = \underline{x}_d$ can be considered as the state error vector remaining on the sliding surface s(e)=0 for all >0. A sufficient condition to achieve this behavior is to select the control strategy such that

$$\frac{1}{2}\frac{d}{dt} s^2(\underline{e}) \leq -\eta |s| \quad , \quad \eta \geq 0 \tag{4}$$

The system is controlled in such a way that the state always moves towards the sliding surface and hits it. The sign of the control value must change at the intersection between the state trajectory and sliding surface.

Consider the control problem of nonlinear systems (1), if $f(\underline{x}, t)$ and $g(\underline{x}, t)$ are known. The SMC input$u^\bullet$ guarantees the sliding condition of (4).

$$u^\bullet = \frac{1}{g(\underline{x}, t)} \left[-\sum_{i=1}^{n-1} c_i e^{(i)} - f(\underline{x}, t) + x_d^{(n)} - \eta \, sng(s) \right] \tag{5}$$

Where

$$sng(s) = \begin{cases} 1 & \text{for} \quad s > 0 \\ 0 & \text{for} \quad s = 0 \\ -1 & \text{for} \quad s < 0 \end{cases} \tag{6}$$

Let the Lyapunov function candidate defined as

$$V_1 = \frac{1}{2} s^2(\underline{e}) \tag{7}$$

Differentiating (7) with respect to time, along the system trajectory as

$$\dot{V} = s.\dot{s} = s.\left(c_1 \dot{e} + c_2 \ddot{e} + \cdots + c_{n-1} e^{(n-1)} + x^{(n)} + x_d^{(n)}\right)$$

$$= s.\left(\sum_{i=1}^{n-1} c_i e^{(i)} + f(\underline{x}, t) + g(\underline{x}, t) u + d(t) - x_d^{(n)} \right) \leq -\eta |s| \tag{8}$$

Hence the SMC inputu• guarantees the sliding condition of (4). It is obvious that in order to satisfy the sliding condition, a hitting control term must be added. i.e. $u^* = u_{eq} - u_{sw}$ where

$$u_{eq} = g(\underline{x},t)^{-1}\left[-\sum_{i=1}^{n-1} c_i e^{(i)} - f(\underline{x},t) + x_d^{(n)}\right] \tag{9}$$

$$u_{sw} = -g(\underline{x},t)^{-1}.\eta sng(s) \tag{10}$$

However, f and g are unknown, it is difficult to apply the control law (5) for an unknown nonlinear plant. Moreover, the switching-type control term u_{sw} will cause chattering problem. To solve these problems, we propose the Adaptive Type-2 Fuzzy Sliding Mode Control Algorithm.

3 – AdaptiveType-2 Fuzzy Sliding Mode Control:

The result in (5) is realizable only while $f(\underline{x},t)$ and $g(\underline{x},t)$ are well known. However, $f(\underline{x},t)$ and $g(\underline{x},t)$ are unknown an the ideal controller (5) cannot be implemented. We replace $f(\underline{x},t)$ and $g(\underline{x},t)$ by the type-2 fuzzy logic system. Moreover, we employ PI control term in order to avoid chattering problem. The input and output of the continuous time PI controller is in the form of:

$$u_p = k_p s + k_i \int s dt \tag{11}$$

k_p and k_i are control gains to be designed. Equation (11) can be rewritten as

$$\hat{p}(s|\underline{\theta}_p) = \underline{\theta}_p^T \psi(s) \tag{12}$$

where $\underline{\theta}_p = [k_p, k_i]^T \in R^2$ is an adjustable parameters during the control produce and $\underline{\psi}^T(s) = \left[s, \int sdt\right] \in R^2$ is a regressive vector. In order to derive the SMC law (5), we use fuzzy logic system to approximate the unknown function $(\underline{x},t)$, $g(\underline{x},t)$ and employ an adaptive PI control term to attenuate chattering action problem and improve steady state performance.

Hence, the resulting control law is as follows:

$$u = \frac{1}{\hat{g}(\underline{x},\underline{\theta}_g)}\left[-\hat{f}(\underline{x},\underline{\theta}_f) - \sum_{i=1}^{n-1} c_i e^i + x_d^{(n)} - \hat{p}(s|\underline{\theta}_p)\right] \tag{13}$$

$$\hat{f}(X,\theta_f) = \frac{1}{2}\theta_f^T(\xi_l(X) + \xi_r(X)) \tag{14}$$

$$\hat{g}(X,\theta_g) = \frac{1}{2}\theta_g^T(\eta_l(X) + \eta_r(X)) \tag{15}$$

In order to avoid the chattering problem, the switching control u_{sw} is replaced by the PI control action when the state is within a boundary layer $|s| < \Phi$ and the control action is kept at the saturated value when the state is outside the boundary layer. Hence, we set $|\hat{p}(s|\underline{\theta}_p)| = D + \eta + \omega_{max}$ when $|s| < \Phi$ where Φ is the thickness of the boundary layer and ω_{max} is the maximum approximation error of the type-2 fuzzy system.

Theorem 1: Consider the control problem of the nonlinear system (1). If control (13) is applied $\hat{f},\hat{g}$, and $\hat{p}$ are given by (14), (15) and (12), the parameters vector $\underline{\theta}_f, \underline{\theta}_g$ and $\underline{\theta}_f, \underline{\theta}_p$ are adjusted by the adaptive law (16) , (17) and (18). The closed loop system signals will be bounded and the tracking error will converge to zero asymptotically.

$$\dot{\underline{\theta}}_f = 0.5\gamma_1 s\left(\xi_l(X) + \xi_r(X)\right) \tag{16}$$

$$\dot{\underline{\theta}}_g = 0.5\gamma_2 s\left(\eta_l(X) + \eta_r(X)\right) \tag{17}$$

$$\dot{\underline{\theta}}_p = \gamma_3 s\underline{\psi}(s) \tag{18}$$

Proof : Define the optimal parameters of type-2 fuzzy systems

$$\underline{\theta}_f^* = \arg\min_{\underline{\theta}_f \in \Omega_f}\left(\sup_{\underline{x}\in R^n}\left|\hat{f}(\underline{x} \mid \underline{\theta}_f) - f(\underline{x}, t)\right|\right) \tag{19}$$

$$\underline{\theta}_g^* = \arg\min_{\underline{\theta}_g \in \Omega_g}\left(\sup_{\underline{x}\in R^n}\left|\hat{g}(\underline{x} \mid \underline{\theta}_g) - g(\underline{x}, t)\right|\right) \tag{20}$$

$$\underline{\theta}_p^* = \arg\min_{\underline{\theta}_p \in \Omega_p}\left(\sup_{\underline{x}\in R^n}\left|\hat{p}(\underline{x} \mid \underline{\theta}_p) - u_{sw}\right|\right) \tag{21}$$

where Ω_f, Ω_g and Ω_p are constraint sets for $\underline{\theta}_f, \underline{\theta}_g$, and $\underline{\theta}_p$, respectively. Define the minimum approximation error.

$$\omega = f(\underline{x}, t) - \hat{f}(\underline{x} \mid \underline{\theta}_f) + \left(g(\underline{x}, t) - \hat{g}(\underline{x} \mid \underline{\theta}_g)\right)u \tag{22}$$

Assumption 1:

$$\Omega_f = \left\{\underline{\theta}_f \in R^n \mid |\underline{\theta}_f| \le M_f\right\}$$

$$\Omega_g = \left\{\underline{\theta}_g \in R^n \mid 0 < \varepsilon \le |\underline{\theta}_g| \le M_g\right\}$$

$$\Omega_p = \left\{\underline{\theta}_p \in R^2 \mid |\underline{\theta}_p| \le M_p\right\}$$

where M_f ε, M_g, and M_p are pre-specified parameters. And assume the fuzzy parameters $\underline{\theta}_f, \underline{\theta}_g$ and the PI control parameter $\underline{\theta}_p$ never reach the boundaries. Then, we have

$$\dot{s} = \sum_{i=1}^{n-1} c_i e^{(i)} + x^{(n)} - x_d^{(n)}$$

$$= \sum_{i=1}^{n-1} c_i e^{(i)} + f(\underline{x}, t) + g(\underline{x}, t)u + d(t) - x_d^{(n)}$$

$$= \sum_{i=1}^{n-1} c_i e^{(i)} + f(\underline{x}, t) - \hat{f}(\underline{x} \mid \underline{\theta}_f) + \left(g(\underline{x}, t) - \hat{g}(\underline{x} \mid \underline{\theta}_g)\right)u - \sum_{i=1}^{n-1} c_i e^{(i)} + x_d^{(n)} - \hat{p}(s \mid \underline{\theta}_p) + d(t) - x_d^{(n)}$$

$$= f(\underline{x}, t) - \hat{f}(\underline{x} \mid \underline{\theta}_f) + \left(g(\underline{x}, t) - \hat{g}(\underline{x} \mid \underline{\theta}_g)\right)u - \hat{p}(s \mid \underline{\theta}_p) + d(t) + \omega$$

$$= \hat{f}(\underline{x} \mid \underline{\theta}_f^*) - \hat{f}(\underline{x} \mid \underline{\theta}_f) + \left(\hat{g}(\underline{x} \mid \underline{\theta}_g^*) - \hat{g}(\underline{x} \mid \underline{\theta}_g)\right)u - \hat{p}(s \mid \underline{\theta}_p) + \hat{p}(s \mid \underline{\theta}_p^*) - \hat{p}(s \mid \underline{\theta}_p^*) + d(t) + \omega$$

$$= 0.5\underline{\phi}_f^T\left(\xi_l(X) + \xi_r(X)\right) + 0.5\underline{\phi}_g^T\left(\eta_l(X) + \eta_r(X)\right).u + \underline{\phi}_p^T\underline{\psi}(s) - \hat{p}(s \mid \underline{\theta}_p^*) + d(t) + \omega$$

Where $\underline{\phi}_f = \underline{\theta}_f^* - \underline{\theta}_f$ $\underline{\phi}_g = \underline{\theta}_g^* - \underline{\theta}_g$ and $\underline{\phi}_p = \underline{\theta}_p^* - \underline{\theta}_p$

Now consider the Lyapunov candidate

$$V = \frac{1}{2}s^2 + \frac{1}{2\gamma_1}\underline{\phi}_f^T\underline{\phi}_f + \frac{1}{2\gamma_2}\underline{\phi}_g^T\underline{\phi}_g + \frac{1}{2\gamma_3}\underline{\phi}_p^T\underline{\phi}_p \quad (24)$$

The time derivative of V along the error trajectory (24) is

$$\dot{V} = s.\dot{s} + \frac{1}{\gamma_1}\underline{\phi}_f^T\underline{\dot{\phi}}_f + \frac{1}{\gamma_2}\underline{\phi}_g^T\underline{\dot{\phi}}_g + \frac{1}{\gamma_3}\underline{\phi}_p^T\underline{\dot{\phi}}_p$$

$$= s\left(0.5\underline{\phi}_f^T(\xi_l(X) + \xi_r(X)) + 0.5\underline{\phi}_g^T(\eta_l(X) + \eta_r(X)).u + \underline{\phi}_p^T\underline{\psi}(s) - \hat{p}(s \mid \underline{\theta}_p^*) + \omega + d(t)\right) + \frac{1}{\gamma_1}\underline{\phi}_f^T\underline{\dot{\phi}}_f + \frac{1}{\gamma_2}\underline{\phi}_g^T\underline{\dot{\phi}}_g + \frac{1}{\gamma_3}\underline{\phi}_p^T\underline{\dot{\phi}}_p$$

$$= 0.5\underline{\phi}_f^T(\xi_l(X) + \xi_r(X)) + \frac{1}{\gamma_1}\underline{\phi}_f^T\underline{\dot{\phi}}_f + 0.5s\underline{\phi}_g^T(\eta_l(X) + \eta_r(X)).u + \frac{1}{\gamma_2}\underline{\phi}_g^T\underline{\dot{\phi}}_g + s\underline{\phi}_p^T\underline{\psi}(s) + \frac{1}{\gamma_3}\underline{\phi}_p^T\underline{\dot{\phi}}_p - s\hat{p}(s \mid \underline{\theta}_p^*) + s\omega + sd(t)$$

$$= \frac{1}{\gamma_1}\underline{\phi}_f^T(0.5\gamma_1 s(\xi_l(X) + \xi_r(X)) + \underline{\dot{\phi}}_f) + \frac{1}{\gamma_2}\underline{\phi}_g^T(0.5\gamma_2 s(\eta_l(X) + \eta_r(X)).u + \underline{\dot{\phi}}_g) + \frac{1}{\gamma_3}\underline{\phi}_p^T\left(s\underline{\psi}(s) + \underline{\dot{\phi}}_p\right) - s\hat{p}(s \mid \underline{\theta}_p^*) + s(\omega + d(t))$$

$$\le \frac{1}{\gamma_1}\underline{\phi}_f^T(0.5\gamma_1 s(\xi_l(X) + \xi_r(X)) + \underline{\dot{\phi}}_f) + \frac{1}{\gamma_2}\underline{\phi}_g^T(0.5\gamma_2 s(\xi_l(X) + \xi_r(X)).u + \underline{\dot{\phi}}_g) + \frac{1}{\gamma_3}\underline{\phi}_p^T\left(s\underline{\psi}(s) + \underline{\dot{\phi}}_p\right) - s(D + \eta)\mathrm{sgn}(s) + sd(t) + s\omega$$

$$< \frac{1}{2\gamma_1}\underline{\phi}_f^T(\gamma_1 s(\xi_l(X) + \xi_r(X)) + 2\underline{\dot{\phi}}_f) + \frac{1}{2\gamma_2}\underline{\phi}_g^T(\gamma_2 s(\xi_l(X) + \xi_r(X)).u + 2\underline{\dot{\phi}}_g) + \frac{1}{\gamma_3}\underline{\phi}_p^T\left(s\underline{\psi}(s) + \underline{\dot{\phi}}_p\right) - \eta|s| + s\omega \quad (25)$$

Where $\underline{\dot{\phi}}_f = -\underline{\dot{\theta}}_f$, $\underline{\dot{\phi}}_g = -\underline{\dot{\theta}}_g$ and $\underline{\dot{\phi}}_p = -\underline{\dot{\theta}}_p$

Substitute (16) , (17) and (18) into (19), then we have

$$\dot{V} \le s\omega - \eta|s| \le 0 \quad (26)$$

since ω it the minimum approximation error, (26) is the best we can obtain. Therefore, all signals in the system are bounded. Obviously, if $\underline{e}(0)$ is bounded, then $\underline{e}(t)$ is also bounded for all. Since the reference signal $\underline{x}_d$ is bounded, then the system states is bounded as well. To complete the proof and establish asymptotic convergence of the tracking error, we need proving that s→0 as t→∞. Assume that $|s| \le \eta_s$ then equation (26) can be rewritten as

$$\dot{V} \le |s||\omega| - |s|\eta \le \eta_s|\omega| - |s|\eta \quad (27)$$

Integrating both sides of (27), we have

$$\int_0^t |s| d\tau \le \frac{1}{\eta}(|V(0)| + V(t)) + \frac{\eta_s}{\eta}\int_0^t |\omega| d\tau \qquad (28)$$

then we have $\in L_1$. Form (26), we know that s is bounded and every term in (23) is bounded. Hence, $s, \dot{s} \in L_\infty$, use of Barbalat's lemma Sastry and Bodson, (1989). We have $s \to 0$ as $t \to \infty$ the system is stable and the error will asymptotically converge to zero. The adaptive fuzzy control system is shown in Fig.1

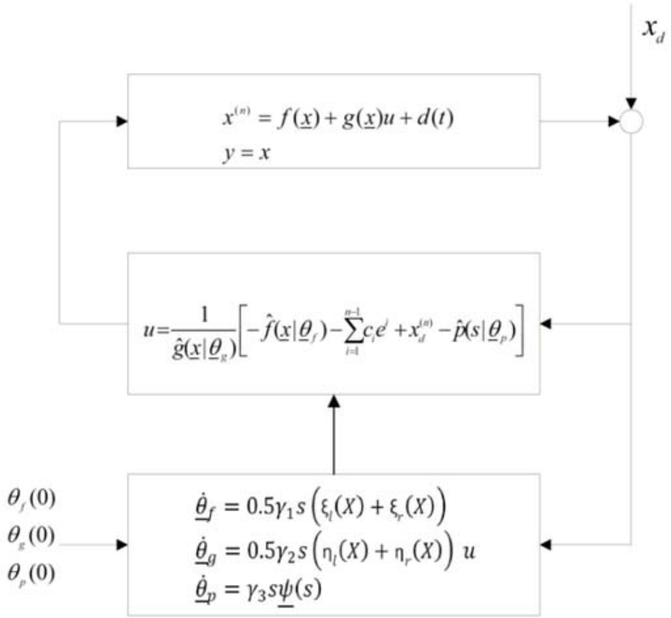

Fig. 1: The block diagram of the proposed controller

Remark 1: The above stability result is achieved under the assumption 1 that all the parameters boundness is ensured. To guarantee the parameters are bounded. The adaptive laws (16), (17) and (18) can be modified by using the projection algorithm (L.X. Wang. 1997), (L.X. Wang. 1993). To summarize the above analysis, the step-by-step procedures for the adaptive fuzzy sliding mode control algorithm is proposed as follow.

Design Procedure:

Step 1. Select proper initial values of PI parameters
Step 2. Specify the desired coefficients c_1, c_2, ... ,c_{n-1} such as in (3).
Step 3. Select the learning coefficients γ_1 ,γ_2 and γ_3

Step 4. Define m_i type-2 fuzzy sets $\tilde{F}_i$ for linguistic variable x_i and the membership functions $\mu_{\tilde{F}_i}$ is uniformly cover the universe of discourse, for I= 1,2, ..., n.
Step 5.Construct the fuzzy rule bases for the fuzzy System $\hat{f}(\underline{x}|\underline{\theta}_f)$ and $\hat{g}(\underline{x}|\underline{\theta}_g)$.

Step 6. Construct the fuzzy systems $\hat{f}(\underline{x}|\underline{\theta}_f) = 0.5\underline{\theta}_f^T\left(\xi_l(X) + \xi_r(X)\right)$ and $\hat{g}(\underline{x}|\underline{\theta}_g) = 0.5\underline{\theta}_g^T\left(\xi_l(X) + \xi_r(X)\right)$.

Step 7. Construct the control law (13) with the adaptive law in (16) , (17) and (18).

Step 8. Obtain the control and apply to the plant, then compute the adaptive law (16) , (17) and (18) to adjust the parameter vector $\underline{\theta}_f, \underline{\theta}_g$, and $\underline{\theta}_p$.

4 – Simulations and comparing two method of typ-1 and type-2:

Now we want to apply proposed tracking controller for inverted pendulum demonstrated in Fig. 2. The control objective is to maintain the system to track the desired angle trajectory, $x_d = \theta_d = \pi/10(\sin(t) + 0.3\sin(3t))$

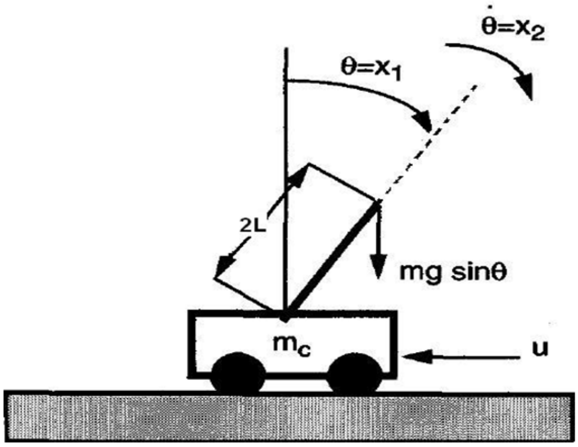

Fig. 2: Inverted pendulum system

The dynamic equations of such system are given by (K. Chafaa, *et al.* 2007).

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{g.\sin x_1 - \dfrac{m.l.x_2^2.\cos x_1.\sin x_1}{m_c + m}}{l\left(\dfrac{4}{3} - \dfrac{m.\cos^2 x_1}{m_c + m}\right)} + \dfrac{\dfrac{\cos x_1}{m_c + m}}{l\left(\dfrac{4}{3} - \dfrac{m.\cos^2 x_1}{m_c + m}\right)} u + d(t) \\ y = x_1 \end{cases} \tag{29}$$

Where x_1 is the angular displacement of the pendulum, x_2 is the angular velocity of the pendulum, m is the mass of pole, m_c is the mass of the cart, I is the half-length from the center of gravity to the pivot and $g = 9.8 m/s^2$ is the acceleration due to gravity, in Simulations; The system parameters are given as $m_c = 1kg$, $m = 0.1kg$, $l = 0.5m$, and is assumed to be a square wave with amplitude ±0.5 and the period 2π. Choose the sliding surface as $s = c_1 e + \dot{e}$ and c_1 = 6. The initial values of parameters $\underline{\theta}_p$ are set by $k_p(0)$=10 and $k_i(0)$ = 20.

Now we consider two following conditions:
1 – There is no uncertainty in system.
2 – There is uncertainty in system.

In froming $\hat{f}$, we suppose :

- For x_1, there is p_1 fuzzy sets than p_1 = 5
- For x_1, there is p_2 fuzzy sets than p_2 = 5

So fuzzy rule base for modeling includes 25 rules

And in froming $\hat{g}$, we suppose:

- For x_1, there is p_1 fuzzy sets than p_1 = 5
- For x_1, there is p_2 fuzzy sets than p_2 = 5

So fuzzy rule base for *g* modeling includes 25 rules.

4.1. consider at the situation when there is no uncertainty in system
4.1.1. tracking control to system modeling by AT1FSMC

We assume that type-1 fuzzy rule base for description g,f is as follows:

$$\begin{cases} if \quad x_1 \; is \; F_1^{l_1} \quad x_2 \; is \; F_1^{l_2} \; then \; \hat{f} \; is \; E^{l_1 l_2} \\ l_1 = 1 \dots, p_1 = 5, l_2 = 1 \dots, p_2 = 5, l_1 l_2 = 1 \dots, p_1 p_2 = 25 \end{cases}$$

$$\begin{cases} if \quad x_1 \; is \; G_1^{l_1} \quad x_2 \; is \; G_1^{l_2} \; then \; \hat{g} \; is \; E^{l_1 l_2} \\ l_1 = 1 \dots, q_1 = 5, l_2 = 1 \dots, q_2 = 5, l_1 l_2 = 1 \dots, q_1 q_2 = 25 \end{cases}$$

And we define Membership Function (MF) in this rule base as follows :

$$\mu_{F_1^1}(z) = \mu_{G_1^1}(z) = e^{-\left(\frac{z+\frac{\pi}{6}}{\frac{\pi}{24}}\right)^2}$$

$$\mu_{F_1^2}(z) = \mu_{G_1^2}(z) = e^{-\left(\frac{z+\frac{\pi}{12}}{\frac{\pi}{24}}\right)^2}$$

$$\mu_{F_1^3}(z) = \mu_{G_1^3}(z) = e^{-\left(\frac{z}{\frac{\pi}{24}}\right)^2}$$

$$\mu_{F_1^4}(z) = \mu_{G_1^4}(z) = e^{-\left(\frac{z-\frac{\pi}{12}}{\frac{\pi}{24}}\right)^2}$$

$$\mu_{F_1^5}(z) = \mu_{G_1^5}(z) = e^{-\left(\frac{z-\frac{\pi}{6}}{\frac{\pi}{24}}\right)^2}$$

Which these functions cover interval of $z = \left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$ as complete and normal. In Fig. 3, above Membership Function is demonstrated on primary variable z which can be x_1 or x_2.

So there are 25 rules for estimating each of f and. The initial consequent parameters of fuzzy are chosen randomly in the interval [0,52]. Let the learning rate $\gamma_1 = 60$, $\gamma_2 = 4$ and $\gamma_3 = 800$. We select initial condition as $\underline{x} = \left[\frac{\pi}{60}, 0\right]^T$.

Moreover, a Gaussian noise with mean zero and variance of 0.025 was injected at the output of the system.

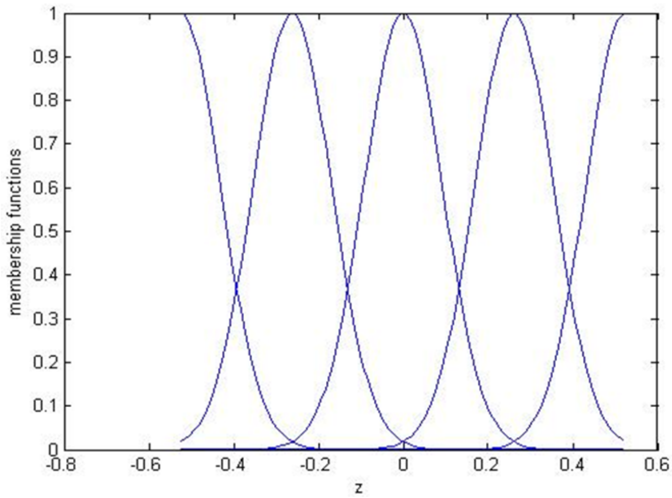

Fig. 3: Fuzzy membership function for modeling inverted pendulum system.

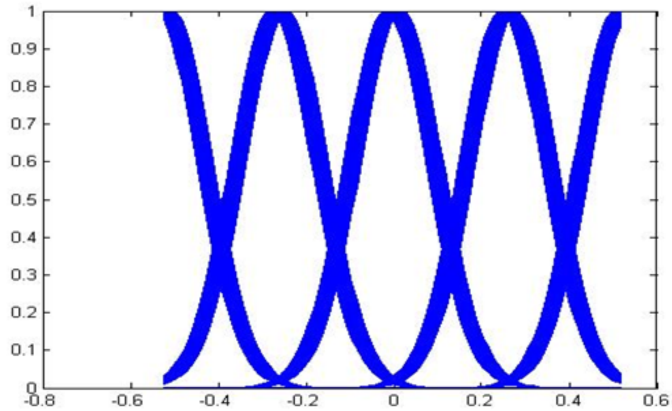

Fig. 4: Show the simulation results for the first condition with type-1 modeling.

We find from figure that tracking performance is good even presence of noise and disturbance.

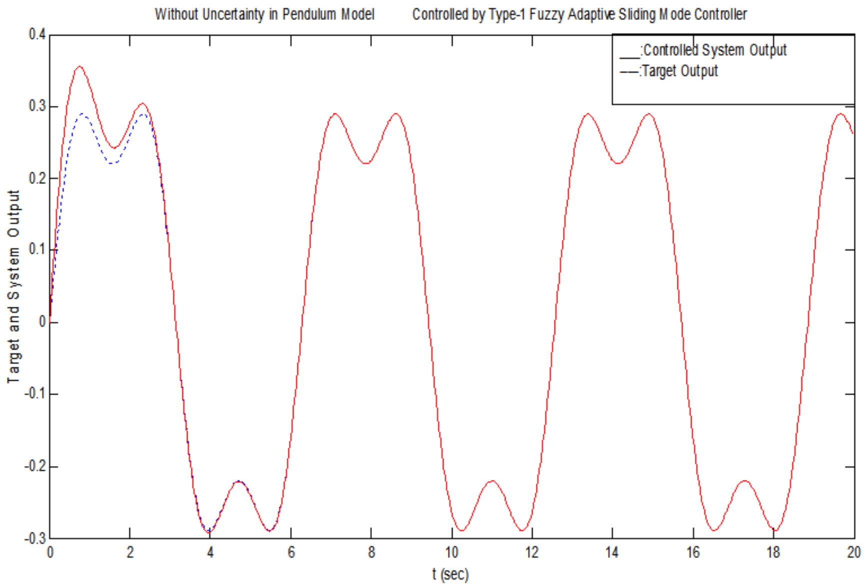

Fig. 5: Desired output and system output (with type-1 modeling)

After beginning, the parameters $\theta_f \in R^{25}$ are adjusted by the adaptive law (16) and the parameters $\theta_g \in R^{25}$ are adjusted by the adaptive law (17). Finally in Fig.4 , it's observed that the closed loop system output ($y=x_1$) will be converged to desired output (y_m).

4.1.2. tracking control to system modeling by AT2FSMC:

We assume that type-1 fuzzy rule base for description g,f is as follows :

$$\begin{cases} if \ \ x_1 \ is \ \ \tilde{F}_1^{l_1} \ \ x_2 \ \ is \ \tilde{F}_1^{l_2} \ then \ \ \hat{f} \ is \ \ \tilde{E}^{l_1 l_2} \\ l_1 = 1 \ldots, p_1 = 5 \ , l_2 = 1 \ldots, p_2 = 5 \ , l_1 l_2 = 1 \ldots, p_1 p_2 = 25 \end{cases}$$

$$\begin{cases} if \ \ x_1 \ is \ \ \tilde{G}_1^{l_1} \ \ x_2 \ \ is \ \tilde{G}_1^{l_2} \ then \ \ \hat{g} \ is \ \ \tilde{E}^{l_1 l_2} \\ l_1 = 1 \ldots, q_1 = 5 \ , l_2 = 1 \ldots, q_2 = 5 \ , l_1 l_2 = 1 \ldots, q_1 q_2 = 25 \end{cases}$$

And we define Membership Function (MF) in this rule base as follows:

$$\mu_{\tilde{F}_1^1}(z) = \mu_{\tilde{G}_1^1}(z) = e^{-\left(\frac{z+\left(\frac{\pi}{6}+\theta\right)}{\frac{\pi}{24}}\right)^2}$$

$$\mu_{\tilde{F}_1^2}(z) = \mu_{\tilde{G}_1^2}(z) = e^{-\left(\frac{z+\left(\frac{\pi}{12}+\theta\right)}{\frac{\pi}{24}}\right)^2}$$

$$\mu_{\tilde{F}_1^3}(z) = \mu_{\tilde{G}_1^3}(z) = e^{-\left(\frac{z+\theta}{\frac{\pi}{24}}\right)^2}$$

$$\mu_{\tilde{F}_1^4}(z) = \mu_{\tilde{G}_1^4}(z) = e^{-\left(\frac{z-\left(\frac{\pi}{12}+\theta\right)}{\frac{\pi}{24}}\right)^2}$$

$$\mu_{\tilde{F}_1^5}(z) = \mu_{\tilde{G}_1^5}(z) = e^{-\left(\frac{z-\left(\frac{\pi}{6}+\theta\right)}{\frac{\pi}{24}}\right)^2}$$

Where $\theta \in [-0.02, 0.02]$

And these functions cover interval of $z = \left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$ as complete and normal. In Fig. 5, above Membership Function is demonstrated on primary variable z which can be x_1 or x_2.

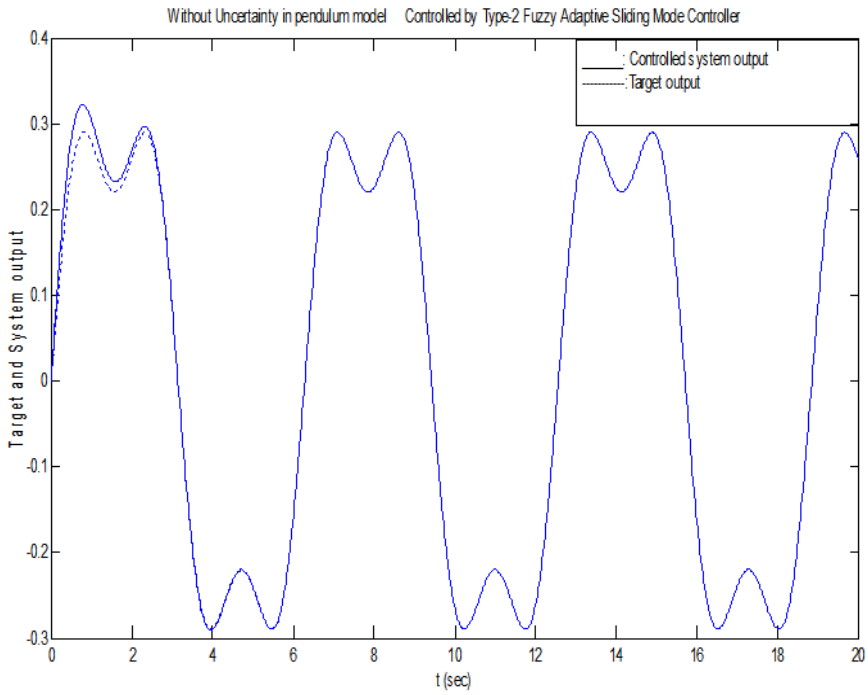

Fig. 6: Interval type-2 fuzzy membership function (with variable average) for modeling inverted pendulum system

By considering previous design parameters, After beginning, the parameters $\theta_f \in R^{25}$ are adjusted by the adaptive law (16) and the parameters $\theta_f \in R^{25}$ are adjusted by the adaptive law (17). Finally in Fig.6, it's observed that the closed loop system output ($y=x_1$) will be converged to desired output (y_m).

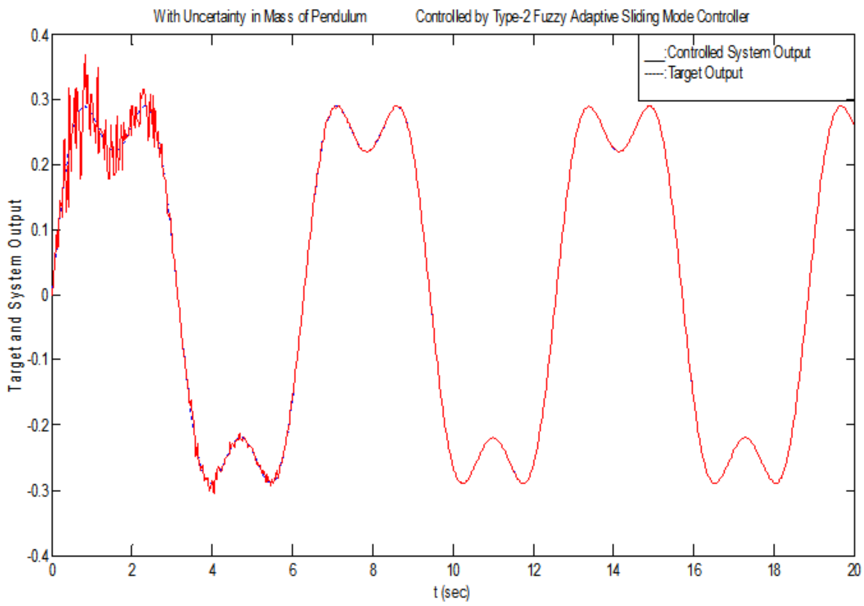

Fig. 7: Desired output and system output (with type-2 modeling)

In table. 1, summary of results in first condition is demonstrated separately according to type of modeling (type-1 and type-2), and for the condition of $X(0) = (\frac{\pi}{60}, 0)^T$. The criterion for comparing is this table is considered Mean of Squares of Error (MSE) between the signals y_m and y in the interval 0 to 20 seconds.

Table. 1: The results obtained in second condition and comparing of two methods with MSE criterion (with uncertainty in the length of pole)

Syestem Modelling	Type-1 Fuzzy	Type-2 Fuzzy (Membership function with variable average)
MSE	27.952e-005	5.8649e-005

4.2. Consider at the Situation When There Is Uncertainty in System:

We examine two following methods separately:

- Applying uncertainty to the mass of pole (*m*).

A Gaussian noise with mean zero and variance of was injected at the mass of pole. It shuld be noted that the mass of pole is exposed to this noise in continuous time.

- Applying uncertainty to the length of pole (*l*).

A Gaussian noise with mean zero and variance of was injected at the length of pole. And the length of $X(0) = (\frac{\pi}{60}, 0)^T$ pole is exposed to this noise in continuous time. It shuld be noted that the length of pole is indefinite without changing its mass.

4.2.1. tracking control to system modeling by AT1FSMC:

4.2.1.1. Applying uncertainty to the mass of pole:

In Fig.7, the manner of tracking of reference signal by the controlled system output with ATIFSMC, for the initial position of is demonstrated. It should be noted that design parameters are assumed the same for all the conditions during simulation.

4.2.1.2. Applying uncertainty to the length of pole:

In Fig.8, the manner of tracking of reference signal by the controlled system output with ATIFSMC, for the initial position of $X(0) = (\frac{\pi}{60}, 0)^T$ is demonstrated.

4.2.2. tracking control to system modeling by AT2FSMC:

In this section, type-2 fuzzy modeling by using MF provided in Fig.5, is done and simulation results for each of uncertainties applied to the amass and the length of pole is demonstrated separately for this MF.

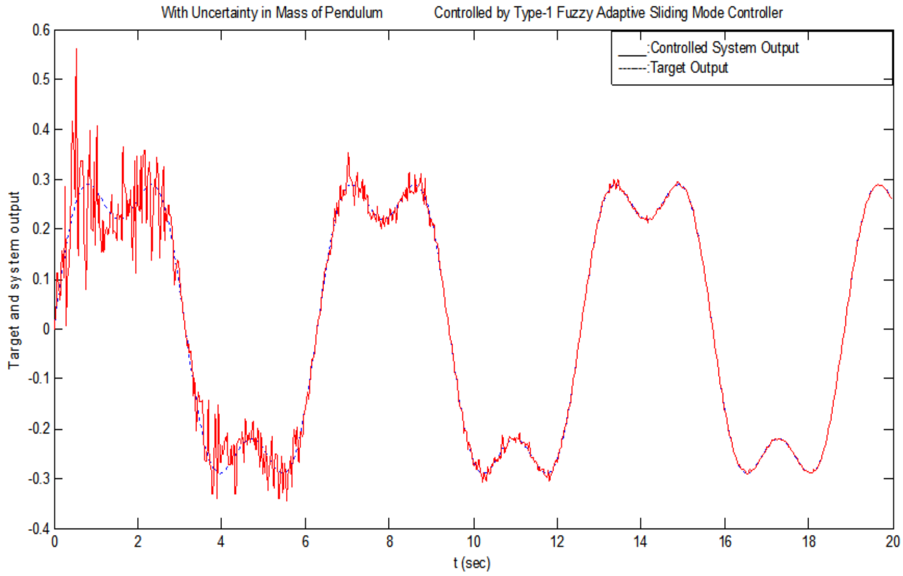

Fig. 8: Desired output and system output (with type-1 modeling)

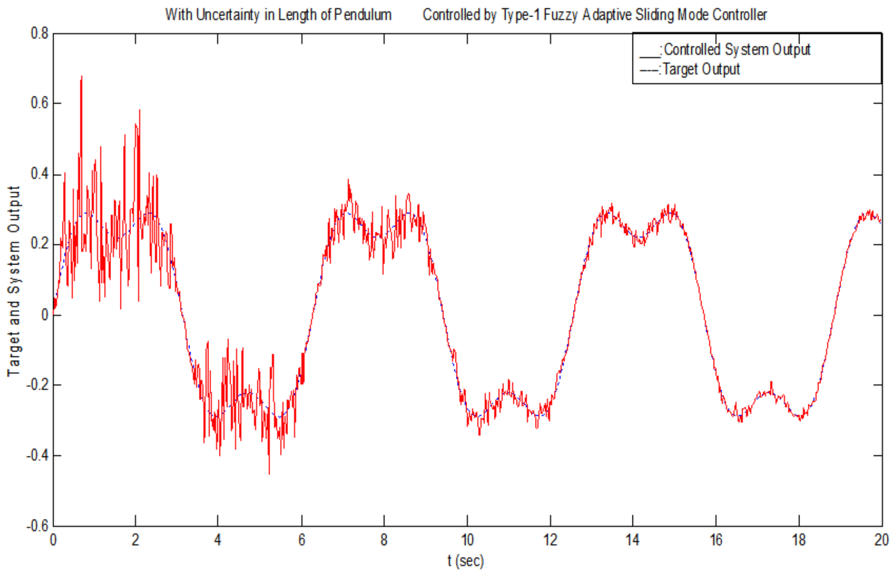

Fig. 9: Desired output and system output (with type-1 modeling)

4.2.2.1. Applying Uncertainty to the Mass of Pole:

In Fig.9, the manner of tracking of reference signal by the controlled system output with AT2FSMC and with interval type-2 fuzzy membership function (with variable average), for the initial position of $X(0) = (\frac{\pi}{60}, 0)^T$ is demonstrated.

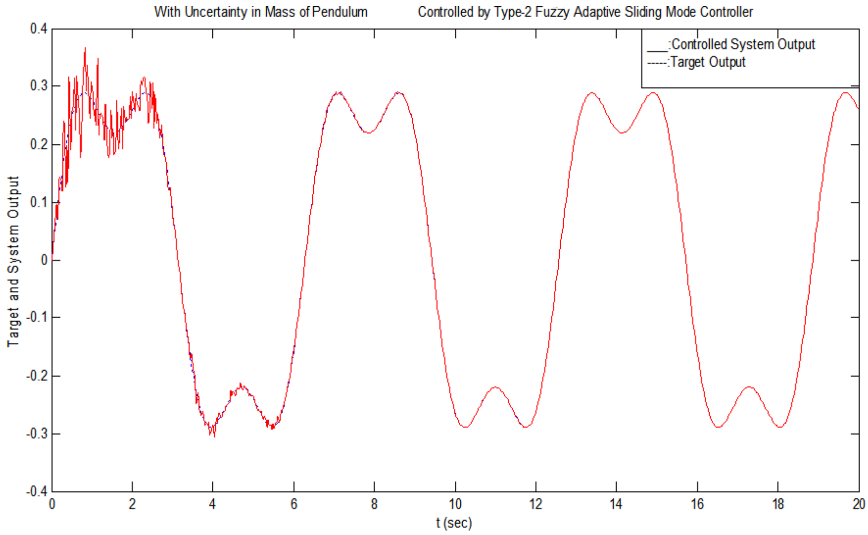

Fig. 10: Desired output and system output (with type-2 modeling)

4.2.2.2. Applying Uncertainty to the Length of Pole:

In Fig.10, the manner of tracking of reference signal by the controlled system output with AT2FSMC and with interval type-2 fuzzy membership function (with variable average), for the initial position of $X(0) = (\frac{\pi}{60}, 0)^T$ is demonstrated.

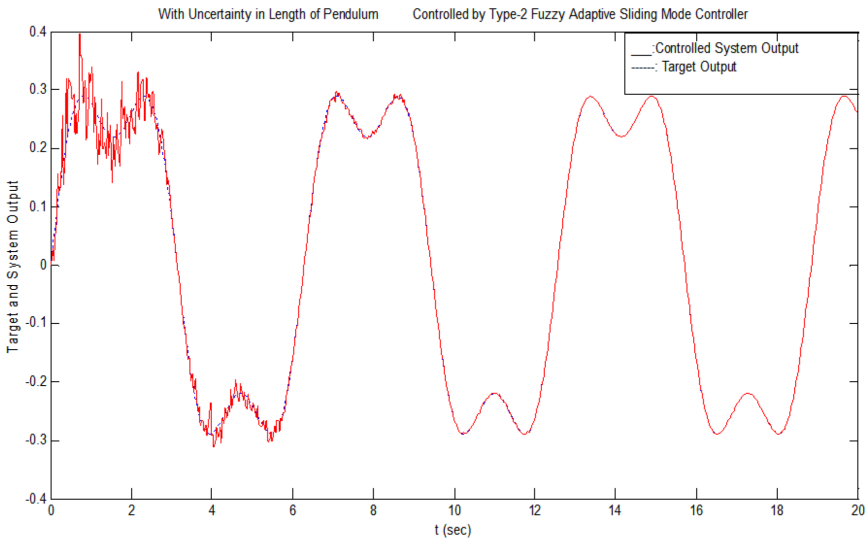

Fig. 11: Desired output and system output (with type-2 modeling)

In table.2, the summary of obtained results for the second condition, separately according to type of modeling (type-1 and type-2) for the condition that uncertainty is applied to the length of pole is shown, and for the condition of $X(0) = (\frac{\pi}{60}, 0)^T$. The criterion for comparing is this table is considered Mean of Squares of Error (MSE) between the signals y_m and in the interval 0 to 20 seconds.

Table. 2: The results obtained in second condition and comparing of two methods with MSE criterion (with uncertainty in the length of pole)

Syestem Modelling	Type-1 Fuzzy	Type-2 Fuzzy(Membership function with variable average)
MSE	0.0035	2.7325e-004

Table. 3: The results obtained in second condition and comparing of two methods with MSE criterion (with uncertainty in the mass of pole)

Syestem Modelling	Type-1 Fuzzy	Type-2 Fuzzy (Membership function with variable average)
MSE 0.0014	1.6362e-004	

In table.3, as same of table.2, the summary of obtained results for the second condition, this time for the condition that uncertainty is applied to the mass of pole is shown.

Conclusions:

In this paper, an adaptive type-2 fuzzy sliding control algorithm has been proposed for a class of unknown nonlinear systems. The proposed control scheme provides good transient and robust performance. Moreover, the proposed controller can be avoided than the chattering phenomenon. The Lyapunov stability theorem has been used to testify to the asymptotic stability of the closed-loop system. As we can see in the tables of the results in the first and second conditions, type-2 fuzzy system with type-2 membership function (T2MF) has better tracking performance than type-1 fuzzy system, either in condition with uncertainty or without uncertainty.

REFERENCES

Chem, B.S., C.H. Lee and Y.C. Chang, 1996. " H tracking design of uncertain nonlinear SISO systems: Adaptive fuzzy approach," IEEE Trans. Fuzzy Syst., 4: 32-43.

Chafaa, K., L. Saidi, L. Ghanai and B. Benmahammed, 2007. "Indirect adaptive interval type-2 fuzzy control for nonlinear systems," International Journal of Modelling, Identification and Control, 2: 106-119.

Kim, S.W. and J.J. Lee, 1995. "Design of a fuzzy controller with fuzzy sliding surface," Fuzzy Sets and Systems, 71(3): 359-367.

Sastry, S.and M. Bodson, 1989. Adaptive Control: Stability, Convergence and Robustness. Prentice Hall.

Slotine, J.E., W.P. Li, 1991. Applied Nonlinear Control, Prentice Hall.

Utkin, V.I., 1977. "Variable structure systems with sliding modes: A survey," IEEE Trans. Automatic Contr., 22: 212-222.

Wang, J., A.B. Rad and P.T. Chan, 2001. "Indirect adaptive fuzzy sliding mode control: Part I-Fuzzy Switching," Fuzzy Sets and Systems, 122(1): 21-30.

Wang, L.X., 1997. A Course in Fuzzy System and Control, Prentice Hall.

Wang, L.X., 1993. "Stable adaptive fuzzy control of nonlinear systems," IEEE Trans. Fuzzy Syst., l(1): 146-155.

Xie, W.F., A.B. Rad, 1999. "Fuzzy on line identification of SISO nonlinear systems," Fuzzy Sets and Systems, 107(3): 323-334.