Australian Journal of Basic and Applied Sciences, 5(3): 11-19, 2011
ISSN 1991-8178

An Optimal Quantity Discount Pricing Policy for Deteriorating Items

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Abstract: We discuss a quantity discount problem between a seller (wholesaler) and a buyer (retailer). The seller purchases products from an upper-leveled supplier (manufacturer) and then sells them to the buyer who faces her/his customers' demand. The seller attempts to increase her/his profit by controlling the buyer's order quantity through a quantity discount strategy and the buyer tries to maximize her/his profit considering the seller's proposal. In this study, we focus on the case where both the seller's and the buyer's inventory levels of the product are continuously depleting due to the combined effects of its demand and deterioration. The deterioration rate is assumed to be a constant fraction of the on-hand inventory. We formulate the above problem as a Stackelberg game between the seller and buyer to analyze the existence of the seller's optimal quantity discount pricing policy which maximizes her/his total profit per unit of time. Numerical examples are presented to illustrate the theoretical underpinnings of the proposed model.

Key words: quantity discount, deterioration items, total profit, Stackelberg game

INTRODUCTION

Quantity discount models between the seller and the buyer have extensively been studied in the inventory literature (Sethi, 1984; Hardly, 1963; Peterson, 1969; Monahan, 1984; Lee, 1984; Data, 1987; Rosenblatt, 1985; Monahan, 1988; Parlar, 1995). Traditional studies on quantity discount models have focused on the problem of determining the buyer's economic order quantity, for a given quantity discount schedule proposed by the seller[1-3]. From the seller's point of view, on the other hand, models on optimal quantity discount pricing policies have also been studied. Monahan (1984), Lee and Rosenblatt (1984) Data and Srikanth (1987) and Rosenblatt and Lee (1985) have formulated the transaction between the seller and the buyer. The framework of transaction is referred to as *buyer-seller transaction*. They discussed methods for determining an optimal discount pricing policy which maximizes the seller's profit. Parlar and Wang (1995) have proposed a model using a game theoretical approach to analyze the quantity discount problem as a perfect information game. They also discussed the case of incomplete information about the buyer's cost structure. These models assumed that both the seller's and the buyer's inventory policies can be described by classical economic order quantity (EOQ) models. The classical EOQ model is a cost-minimization inventory model with a constant demand rate. It is one of the most successful models in all the inventory theories due to its simplicity and easiness.

In many real-life situations, items deteriorate continuously such as medicine, volatile liquids, blood banks (Changa, 2006). Dave and Patel (1981) discussed an inventory model for deterioration items when shortages were not allowed. Sachan (1984) then extended their model to allow for shortages. Hariga (1996) developed optimal EOQ models with log-concave demand for deterioration items under three replenishment policies. Yang *et al.* (2001) provided various inventory models with time-varying demand patterns under inflation. For more work: see also (Changa, 2006; Dave, 1981; Sachan, 1984; Hariga, 1996; Yang, 2001; Goyal, 2001). Yang (2004) has developed the model to determine an optimal pricing and a ordering policy for deteriorating items with quantity discount which is proposed by the vendor. However, his model assumed the deteriorating rate of the product at the vendor's store to be zero.

In this study, we discuss a quantity discount problem between a seller (wholesaler) and a buyer (retailer) under circumstances where both the wholesaler's and the retailer's inventory levels of the product are continuously depleting due to the combined effects of its demand and deterioration. The wholesaler purchases products from an upper-leveled supplier (manufacturer) and then sells them to the retailer who faces her/his customers' demand. The wholesaler is interested in increasing her/his profit by controlling the retailer's order quantity through the quantity discount strategy. The retailer attempts to maximize her/his profit considering the

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wholesaler's proposal. We formulate the above problem as a Stackelberg game between the wholesaler and the retailer to show the existence of the wholesaler's optimal quantity discount pricing policy which maximizes her/his total profit per unit of time. Numerical examples are presented to illustrate the theoretical underpinnings of the proposed model.

II. Notations and Assumptions:

The wholesaler uses a *quantity discount strategy* in order to improve her/his profit. The wholesaler proposes, for the retailer, an order quantity per lot along with the corresponding discounted wholesale price, which induces the retailer to alter her/his replenishing policy.
We consider the two options throughout the present study as follows:

Option V_1:

The retailer does not adopt the quantity discount proposed by the wholesaler. When the retailer chooses this option, she/he purchases the products from the wholesaler at an initial price in the absence of the discount, and she/he determines her/himself an optimal order quantity which maximizes her/his own total profit per unit of time.

Option V_2:

The retailer accepts the quantity discount proposed by the wholesaler.
The main notations used in this paper are listed below:
c_s: the wholesaler's unit acquisition cost (unit purchasing cost from the upper-leveled manufacturer).
p_s: the wholesaler's initial unit selling price, i.e., the retailer's unit acquisition cost in the absence of the discount.
p_b: the retailer's unit selling price, i.e., unit purchasing price for her/his customers.
a_s, a_b: the wholesaler's and the retailer's ordering cost per lot, respectively.
h_s, h_b: the wholesaler's and the retailer's inventory holding cost per item and unit of time, respectively.
y: the discount rate for the wholesale price proposed by the wholesaler, i.e., the wholesaler offers a unit discounted price of $(1-y)p_s$ with .
Q_i: the retailer's order quantity per lot under Option V_i (i = 1,2).
S_i: the wholesaler's order quantity per lot under Option V_i (i = 1,2).
T_i: the length of the retailer's order cycle under Option V_i (i = 1,2, $T_1 \leq T_2$).
q_s, q_b: the deteriorating rates at wholesaler's store and the retailer's store, respectively ($q_s < q_b$).
m: the constant demand rate of the product

The assumptions in this study are as follows:

i) Both the wholesaler's and the retailer's inventory levels of the product are continuously depleting due to the combined effects of its demand and deterioration.
ii) The rate of replenishment is infinite and the delivery is instantaneous.
iii) Backlogging and shortage are not allowed.
iv) The quantity of the item can be treated as continuous for simplicity.
V) Both the wholesaler and the retailer are rational and use only pure strategies.

The length of the wholesaler's order cycle is given by N_i T_i (N_i = 1, 2) under Option V_i (i = 1,2), where N_i is a positive integer. This is because the wholesaler can possibly reduce the total ordering cost by ordering N_i times of the retailers ordering quantity Q_i (added to amount of items which will be discarded during [0, N_i T_i) due to deterioration) at one time.

III. Retailer's Total Profit:

This section formulates the retailer's total profit per unit of time for the Option V_1 and V_2 available to the retailer.

A. Under Option V_1:

Since the inventory is depleted due to the combined effect of its demand and deterioration, the inventory level, I(t), at time t during $[0,T_1)$ can be expressed by the following differential equation:

$$.dI(t)/dt = -\theta_b I(t) - \mu \qquad (1)$$

By solving the differential equation in (1) with a boundary condition $I(T_1) = 0$ the retailer's inventory level at time t is given by

$$I(t) = \rho\left[e^{\theta_b(T_1 - t)} - 1\right] \tag{2}$$

where $\rho = \mu / \theta_b$.

Therefore, the initial inventory level in the order cycle becomes

$$Q_1 = I(0) = \rho\left(e^{\theta_b T_1} - 1\right) \tag{3}$$

On the other hand, the cumulative inventory, A(T), held during $[0,T_1)$ is expressed by

$$A(T) = \int_0^{T_1} I(t)dt = \frac{\rho}{\theta_b}\left(e^{\theta_b T_1} - 1\right) - \rho T_1 \tag{4}$$

Hence, the retailer's total profit per unit of time under Option V_1 is given by

$$\pi_1(T_1) = \frac{p_b \mu T_1 - p_s Q_1 - h_b A(T_1) - a_b}{T_1}$$

$$= \rho(p_b \theta_1 + h_b) - \frac{\rho\left(p_s + h_b / \theta_b\right)\left(e^{\theta_b T_1} - 1\right) + a_b}{T_1} \tag{5}$$

In the following, the results of analysis are briefly summarized:
Transition of inventory level (N_i = 3)

There exists a unique finite $T_1 = T_1^*$ (>0) which maximizes the π (T_1) in (Lee, 1984). The optimal order quantity is therefore given by

$$Q_1^* = \rho\left(e^{\theta_b T_1^*} - 1\right) \tag{6}$$

The total profit per unit of time becomes

$$\pi_1^* = \rho\left[(p_b \theta_b + h_b) - \theta_b\left(p_s + h_b / \theta_b\right)e^{\theta_b T_1^*}\right] \tag{7}$$

If the retailer chooses Option V_1, her/his order quantity per lot and her/his unit acquisition cost are respectively given by Q_1, and p_s, where p_s is the unit initial price in the absence of the discount. In this case, she/he determines herself the optimal order quantity $Q_1 = Q_1^*$, so that she/he can maximize her/his total profit per unit of time.

B. Under Option V_2:

If the retailer chooses Option V_2, the order quantity and unit discounted wholesale price are respectively given by Q_2 = $\left(e^{\theta_b T_2} - 1\right)$ and $(1-y)p_s$. The retailer's total profit per unit of time can therefore be expressed by

$$\pi_2(T_2, y) = \rho(p_b \theta_b + h_b) - \frac{\rho\left[(1-y)p_s + h_b / \theta_b\right]\left(e^{\theta_b T_2} - 1\right) + a_b}{T_2} \tag{8}$$

IV. Wholesaler's Total Profit:

This section formulates the wholesaler's total profit per unit of time, which depends on the retailer's decision. Figure 1 shows both the wholesaler's and the retailer's transition of inventory level in the case of $N_i = 3$. Figure 1 reveals that the retailer orders Q_i units when her/his inventory level reaches zero, and on the other hand, the wholesaler orders S_i ($\geq Q_i$) units per lot. Remind that S_i is not equal to the N_i Q_i since her/his inventory is also depleting due to deterioration.

A. Total Profit under Option V_1:

If the retailer chooses Option V_1, her/his order quantity per lot and unit acquisition cost are given by Q_1 and p_s, respectively. The length of the wholesaler's order cycle can be divided into N_1 shipping cycles (N_1=1,2,3,…) under Option V_1 as described in assumption vi), where N_1 is also a decision variable for the wholesaler.

The wholesaler's inventory is depleting only due to deterioration during [(j-1)T_1, jT_1) in jth shipping cycle (j = 1,2,…, N_1). Therefore, the wholesaler's inventory level, $I_s(t)$, at time t can be expressed by the following differential equation:

$$dI_s(t)/dt = -\theta_s I_s(t) \tag{9}$$

with a boundary condition $I_s(jT_1) = z_j(T_1)$ where $z_j(T_1)$ denotes the remaining inventory at the end of the jth shipping cycle. By solving the differential equation in (9), the wholesaler's inventory level, $I_s(t)$, at time t is given by

$$I_s^{(j)}(t) = z_j(T_1)e^{\theta_s(jT_1-t)} \tag{10}$$

The wholesaler's cumulative inventory, $B_j(T_1)$, held during jth shipping cycle is expressed by

$$B_j(jT_1) = \int_{(j-1)T_i}^{jT_i} I_s^{(j)}(t)dt = \frac{z_j(T_1)}{\theta_s}\left(e^{\theta_s T_1} - 1\right) \tag{11}$$

It can easily be confirmed that the inventory level at the end of the (N_1-1)th shipping cycle becomes Q_1, i.e. $z_{N_1-1}(T_1) = Q_1$ as also shown in Fig. 1. By induction, we have

$$z_j(T_1) = Q_1 \frac{\gamma_s^{N_1-j}(T_1) - 1}{\gamma_s(T_1) - 1} \tag{12}$$

where

$$\gamma_s(T_1) = e^{\theta_s T_1} \tag{13}$$

The wholesaler's order quantity per lot is then given by

$$S_1 = S_1(T_1) = z_0(T_1) \tag{14}$$

On the other hand, the wholesaler's inventory holding cost during [0, N_1T_1) becomes

$$B(N_1, T_1) = \sum_{j=1}^{N_1-1} B_j(T_1) \tag{15}$$

Hence, for a given N_1, the wholesaler's total profit per unit of time under Option V_1 is given by

$$P_1(N_1, T_1^*) = \frac{N_1 p_s Q_1(T_1^*) - c_s S_1(T_1^*) - h_s B(N_1, T_1^*) - a_s}{N_1 T_1} = \frac{\rho\left(e^{\theta_b T_1^*} - 1\right)\cdot\left[N_1 p_s - \varsigma(N_1, T_1^*)\right] - a_s}{N_1 T_1} \tag{16}$$

where

$$\varsigma(N_1,T_1^*)=c_s\frac{\gamma_s^{N_1}(T_1^*)-1}{\gamma_s(T_1^*)-1}+\frac{h_s}{\theta_s}\left\{\frac{\gamma_s(T_1^*)\left[\gamma_s^{N_1-1}(T_1^*)-1\right]}{\gamma_s(T_1^*)-1}-(N_1-1)\right\} \tag{17}$$

B. Total Profit under Option V_2:

When the retailer chooses Option V_2, she/he purchases Q_2 units at the unit discounted wholesale price (1-y)p_s. In this case, the wholesaler's order quantity per lot under Option V_2, is expressed as $S_2 = S_2(T_2)$, accordingly the wholesaler's total profit per unit of time under Option V_2 is given by

$$P_2(N_2,T_2,y)=\frac{1}{N_2T_2}\left[N_2(1-y)p_sQ_2(T_2)-c_sS_2(T_2)\right.$$
$$\left.-h_sB(N_2,T_2)-a_s\right]=\frac{Q_2(T_2)\left[N_2\cdot(1-y)p_s-\varsigma(N_2,T_2)\right]-a_s}{N_2T_2} \tag{18}$$

where

$$Q_2(T_2)=\rho\left(e^{\theta_sT_2}-1\right) \tag{19}$$

$$S_2(T_2)=z_0(T_2) \tag{20}$$

V. Retailer's Optimal Response:

This section discusses the retailer's optimal response. The retailer prefers Option V_1 over Option V_2 if $\pi_{1'}^* > \pi_2(T_2,y)$ but when $\pi_{1'}^* < \pi_2(T_2,y)$ she/he prefers V_2 to V_1. The retailer is indifferent between the two options if $\pi_{1'}^* = \pi_2(T_2,y)$ which is equivalent to

$$.y=\frac{1}{p_s}\cdot\frac{(p_s+h_b/\theta_b)\left[Q_2(T_2)-\rho\theta_bT_2e^{\theta_bT_1^*}\right]+a_b}{Q_2(T_2)} \tag{21}$$

Let us denote, by $\psi(T_2)$ the right-hand-side of (21). It can easily be shown from (21) that $\psi(T_2)$ is increasing in T_2 ($\geq T_1^*$).

***VI. Wholesaler's Optimal Policy*:**

The wholesaler's optimal values for T_2 and y can be obtained by maximizing her/his total profit per unit of time considering the retailer's optimal response which was discussed in Section V.

Henceforth, let Ω_i (i =1,2) be defined by

$$,\Omega_1=\{(T_2,y)\mid y\leq\psi(T_2)\}$$

$$\Omega_2=\{(T_2,y)\mid y\geq\psi(T_2)\}$$

Figure 2 depicts the region of Ω_i (i =1,2) on the (T_2,y) plane.

A. Under Option V_1:

If $(T_2,y)\in\Omega_1\setminus\Omega_2$ in Fig. 2, the retailer will naturally select Option V_1. In this case, the wholesaler can maximize her/his total profit per unit of time independently of T_2 and y on the condition of $(T_2, y) \in \Omega_1 \setminus \Omega_2$. Hence, for a given N_1, the wholesaler's locally maximum total profit per unit of time in $\Omega_1 \setminus \Omega_2$ becomes

$$P_1^* = P_1(N_1, T_1^*) \tag{22}$$

B. Under Option V_2:

On the other hand, if $(T_2, y) \in \Omega_2 \setminus \Omega_1$ the retailer's optimal response is to choose Option V_2. Then the wholesaler's locally maximum total profit per unit of time in $\Omega_2 \setminus \Omega_1$ is given by

$$P_2^* = \max_{N_2 \in N} \hat{P}_2(N_2) \tag{23}$$

where N signifies the set of positive integers, and

$$\hat{P}_2(N_2) = \max_{(T_2, y) \in \Omega_2 \setminus \Omega_1} P_2(N_2, T_2, y) \tag{24}$$

More precisely, we should use "sup" instead of "max" in (23).

For a given N_2, we show below the existence of the wholesaler's optimal quantity discount pricing policy $(T_2, y) = (T_2^*, y^*)$ which attains (24).

It is easily proven that $P_2 (N_2, T_2, Y)$ in (18) is strictly decreasing in y, and consequently the wholesaler can attain $\hat{P}_2(N_2)$ in (24) by letting .

By letting $y = \psi(T_2)$ in (18), the total profit per unit of time on $y = \psi(T_2)$ becomes

$$P_2(N_2, T_2) = (p_s + h_b / \theta_b)\rho\theta_b e^{\theta_b T_1^*} - \frac{1}{N_2 T_2}\left\{(a_b N_2 + a_s) + Q_2(T_2)\left[C\frac{\gamma_s^{N_2}(T_2) - 1}{\gamma_s(T_2) - 1} - H(N_2)\right]\right\} \tag{25}$$

where

$$C = (c_s + h_s / \theta_s) \tag{26}$$

$$H(N_2) = (h_s / \theta_s - h_b / \theta_b) N_2 \tag{27}$$

Let us here define

$$L(N_2, T_2) \equiv \left[\rho\theta_b T_2 e^{\theta_b T_2} - Q_2(T_2)\right] \cdot \left[C\frac{\gamma_s^{N_2}(T_2) - 1}{\gamma_s(T_2) - 1} - H(N_2)\right]$$

$$+ \frac{Q_2(T_2^*) T_2 \theta_s}{\left[\gamma_s(T_2) - 1\right]^2}\left\{N_2 \gamma_s^{N_2}(T_2)\left[\gamma_s(T_2) - 1\right] - \gamma_s(T_2)\left[\gamma_s^{N_2}(T_2) - 1\right]\right\} \tag{28}$$

We here summarize the result of analysis in relation to the optimal quantity discount policy (T_2^*, y^*) which attains $\hat{P}_2(N_2)$ in (24) when N_2 is fixed to a suitable value.

:$\{N_2 = 1\}$

In this case, there exists a unique finite $\tilde{T}_2$ $(> T_1^*)$ which maximizes $P_2(N_2, T_2)$ in (25), and therefore (T_2^*, y^*) is given by

$$(T_2^*, y^*) \to (\tilde{T}_2, \psi(\tilde{T}_2)) \tag{29}$$

The total profit per unit of time becomes

$$\hat{P}_2(N_2) = \rho\theta_b \left\{ (p_s + h_b/\theta_b)e^{\theta_b T_1^*} - (c_s + h_b/\theta_b)e^{\theta_b T_2^*} \right\} \tag{30}$$

ii) $\{N_2 \geq 2\}$:

In this case, $\tilde{T}_2$ is the unique solution (if it exists) to

$$L(N_2, T_2) = (a_b N_2 + a_s) \tag{31}$$

The optimal quantity discount pricing policy is given by (29). The wholesaler's total profit per unit of time is expressed by

$$\hat{P}_2(N_2) = (p_s + h_b/\theta_b)\rho\theta_b e^{\theta_b T_1^*} - \frac{1}{N_2}\left\{ \rho\theta_b e^{\theta_b T_2^*}\left[C\frac{\gamma_s^{N_2}(T_2^*) - 1}{\gamma_s(T_2^*) - 1} - H(N_2) \right] + \frac{Q_2(T_2^*)\theta_s}{\left[\gamma_s(T_2^*) - 1\right]^2}\left\{ N_2\gamma_s^{N_2}(T_2^*)\left[\gamma_s(T_2^*) - 1\right] \right. \right. \tag{32}$$

C. Under Option V_1 and V_2:

In the case of $(T_2, y) \in \Omega_1 \cap \Omega_2$, the retailer is indifferent between Option V_1 and V_2. For this reason, this study confines itself to a situation where the wholesaler does not use a discount pricing policy $(T_2, y) \in \Omega_1 \cap \Omega_2$

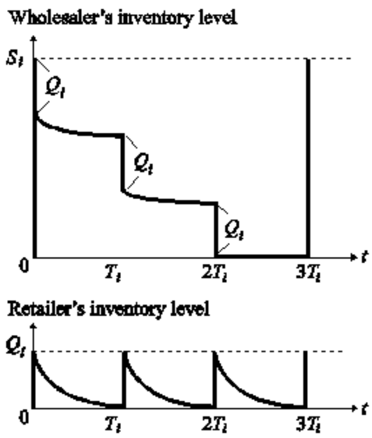

Fig. 1: Transition of inventory level (Ni = 3).

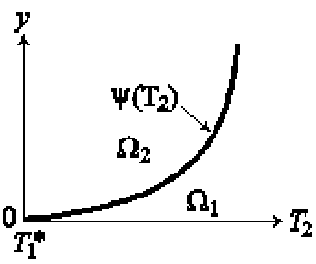

Fig. 1: Characterization of retailer's optimal responses.

VII. Numerical Examples:

Table 1 reveals the results of sensitively analysis in reference to Q_1*, p_1 (= p_s), S_1* = z_0(T_1*), N_1*, P_1*, Q_2*, P_2* = (1-y* p_s), S_2* z_0(T_1*), N_2*, P_2* for (c_s, p_s, p_b, a_b, h_s, h_b, θ_s θ_b, μ) = (100, 300, 600, 1200, 1, 1.1, 0.01, 0.015, 5) when K = 500, 1000, 2000, and 3000.

In Table 1(a), we can observe that both S_1* and N_1* are non-decreasing in a_s. This is because, under Option V_1, the retailer does not adopt the quantity discount proposed by the wholesaler, and therefore the wholesaler should slash her/his total ordering cost by means of ordering at intervals of N_1 times of the retailer's order cycle when her/his ordering cost per lot becomes large.

It is seen in Table 1(b) that, under Option V_2, S_2* increases with a_s, in contrast, N_2* takes a constant value, i.e., we have N_2* = 1. Under Option V_2, the wholesaler can control the retailer's order quantity through the quantity discount. The wholesaler intends to increase her/his order quantity (if necessary) by stimulating the retailer to alter her/his order quantity per lot rather than by increasing N_i since the wholesaler's order quantity per lot jumps up at the moment when N_i increases one step, due to the integer number of N_i. We can also notice in Table 1 that we have P_1* < P_2*.

Table 1: *Sensitivity analysis*

(a) Under Option V_1

a_s	Q_1*	p_1	S_1* (N_1*)	P_1*
500	47.35	300	47.35 (1)	1012.99
1000	47.35	300	99.09 (2)	963.48
2000	47.35	300	99.09 (2)	907.01
3000	47.35	300	155.61 (3)	854.45

(b) Under Option V_2

a_s	Q_2*	p_2*	S_2* (N_2*)	P_2*
500	84.07	292.61	84.07(1)	1046.59
1000	96.13	288.81	96.13 (1)	1015.23
2000	116.94	281.87	116.94 (1)	961.13
3000	134.94	275.77	134.94 (1)	914.32

Conclusions:

In this study, we have discussed a quantity discount problem between a wholesaler and a retailer, particularly when both the wholesaler's and the retailer's inventory levels of the product are continuously depleting due to the combined effects of its demand and deterioration. The wholesaler is interested in increasing her/his profit by controlling the retailer's order quantity thorough the quantity discount strategy. The retailer attempts to maximize her/his profit considering the wholesaler's proposal. We formulate the above problem as a Stackelberg game between the wholesaler and the retailer to show the existence of the wholesaler's optimal quantity discount pricing policy that maximizes her/his total profit per unit of time. We first show the retailer's optimal response, and then clarify the existence of the wholesaler's optimal quantity discount pricing policy.

This study assumes the inventory holding cost to be independent of the purchase cost of the item. In the real circumstances, however, the inventory holding cost depends on its purchase cost, and then it should be expressed in terms of a percentage of the item value. Taking account of such factors is an interesting extension.

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