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Ranking Fuzzy Numbers by Using Radius of Gyration

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Abstract: Ranking fuzzy numbers plays a very important role in linguistic decision making and some other fuzzy application systems. Many methods have been proposed to deal with ranking fuzzy numbers. Recently, Deng and his colleagues, presented a method to rank fuzzy numbers. They employed radius of gyration rank fuzzy numbers; however there were some problems with the ranking method. In this paper, we first indicate the problems of ROG (Radius of Gyration) method and then propose a revised method which can avoid these problems for ranking fuzzy numbers. Since the revised method is based on the ROG method, it is easy to rank fuzzy numbers in a way similar to the original method.

Key words: fuzzy number, ranking, radius of gyration.

INTRODUCTION

Ranking fuzzy numbers is important in decision-making, data analysis, artificial intel- ligence, Economic systems and operations research. Jain (Jain, 1976; 1978), Dubois and Prade (1978) introduced the relevant concepts of fuzzy numbers. Bortolan and Degani (1985) reviewed some methods to rank fuzzy numbers, Chen and Hwang (1992) proposed fuzzy multiple attribute decision making, Choobineh and Li (1993) proposed an index for ordering fuzzy numbers, Dias (1993) ranked alternatives by ordering fuzzy numbers, Lee *et al.* (1994) ranked fuzzy numbers with a satisfaction function, Requena *et al.* (1994) utilized artificial neural networks for the automatic ranking of fuzzy numbers, Fortemps and Roubens (1996) presented ranking and defuzzication methods based on area compensation, and Raj *et al.* (1999) investigated maximizing and minimizing sets to rank fuzzy alternatives with fuzzy weights. However, Chu and Tsao (2004) proposed a method of ranking fuzzy numbers with an area between the centroid and original points. Chu and Tsao's method originated from the concepts of Lee, Li (1988) and Cheng (1998). Lee and Li proposed the comparison of fuzzy numbers, for which they considered mean and standard deviation values for fuzzy numbers based on the uniform and proportional probability distributions. Wang and Lee (2008) presented a new method of ranking fuzzy numbers with using radius of gyration. In this paper, we shall propose a new method for ranking fuzzy numbers to overcome the shortcomings of some previous methods. We prepare our discussion in 5 sections.

In Section 2, we give some definitions and preliminaries. In Section 3, we describe radius of gyration method. In Section 4, we first give some examples to state the shortcomings of the previous methods and then present a new method to overcome these problems. More-over, we shall compare our method with some ranking methods of fuzzy numbers. Finally, we conclude in Section 5.

2. Preliminaries:

Here, we review some basic notations of fuzzy sets (taken from (Wang and Lee, 2008)).

Definition 2.1.

Let U be a universe set. A fuzzy set A of U is defined by a membership function $\mu_A \rightarrow [0, 1]$, where $\mu_A(x), \forall x \in U$, indicates the degree of x in A.

Definition 2.2.

A fuzzy subset A of universe set U is normal if and only if $\sup_{x \in U} \mu_A(x) = 1$, where U is the universe set.

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Definition 2.3.

A fuzzy subset A of universe set U is convex if and only if $\mu_A(\lambda x + (1 - \lambda)y) \geq \min\{\mu_A(x), \mu_A(y)\}, \forall x,y \in U, \forall \lambda \in [0,1]$.

Definition 2.4.

A fuzzy set A is a fuzzy number if and only if A is normal and convex on U.

Definition 2.5.

A triangular fuzzy number A is a fuzzy number with a piecewise linear membership function μ_A defined by:

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ \frac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3, \\ 0, & \text{otherwise}, \end{cases}$$

which can be denoted as a triplet (a_1, a_2, a_3).

Definition 2.6.

A trapezoidal fuzzy number A is a fuzzy number with a membership func- tion μA defined by:

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ 1, & a_2 \leq x \leq a_3, \\ \frac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4, \\ 0, & \text{otherwise}, \end{cases}$$

which can be denoted as a quartet (a_1, a_2, a_3, a_4).

The Radius of Gyration of Fuzzy Numbers:

Radius of gyration is a concept in mechanics. Deng and his colleagues gave a new area method to rank fuzzy numbers with the radius of gyration (ROG) points (see in (Deng *et al.,* 2006)). Here, we keep all discussions of them.

The moment of inertia of the area A with respect to the x axis, and the moment of inertia of the area A with respect to the y axis are defined, respectively, as

$$I_x = \int_A y^2 dA, \quad I_y = \int_A x^2 dA. \tag{1}$$

The radius of gyration of an area A with respect to the x axis is defined as the quantity r_x, that satisfies the relation,

$$I_x = r_x^2 A, \tag{2}$$

where I_x is the moment of inertia of A with respect to the x axis. Solving equation for r_x, concludes that

$$r_x = \sqrt{\frac{I_x}{A}} \tag{3}$$

In a similar way, they define the radius of gyration of an area A with respect to the y axis is

$$I_y = r_y^2 A, \tag{4}$$

$$r_y = \sqrt{\frac{I_y}{A}}. \tag{5}$$

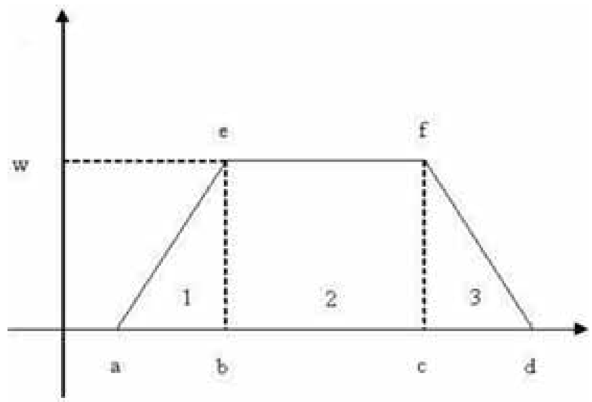

Fig. 1:

They mentioned when a generalized fuzzy number A is given, the radius of gyration (ROG) points of the generalized fuzzy number A is denoted as (r_x (A), r_y (A)) whose value can be obtained by equations (3) and (5). For an area made up of a number of simple shapes, the moment of inertia of the entire area is the sum of the moments of inertia of each of the individual area about the axis desired. For example, the moment of inertia of the generalized trapezoidal fuzzy number in Figure 1 can be obtained as follows:

$$I_x = I_{x1} + I_{x2} + I_{x3} \ , \ I_y = I_{y1} + I_{y2} + I_{y3}. \tag{6}$$

Example 1:

Determine the moment of inertia and the radius of gyration of the generalized trapezoidal fuzzy number. First, the trapezoid (*a, e, f, d*) can be divided into three parts, (*a, e, b*), (*b, e, f, c*), and (*c, f, d*). The moment of inertia of the area (aeb) with respect to x axis, and the moment of inertia of the area (*a, e, b*) with respect to y axis can be calculated

$$\begin{aligned}(I_x)_1 &= \int_{aeb} y^2 dA = \int_0^w y^2 . \frac{(b-a)(w-y)}{w} dy = \frac{(b-a)w^3}{12} \\ (I_y)_1 &= \int_{aeb} x^2 dA = \frac{(b-a)w^3}{4} + \frac{(b-a)a^2 w}{2} + \frac{2(b-a)w^2}{3}\end{aligned} \tag{7}$$

The moment of inertia of area befc and cfd, with respect to x axis and y axis, can be obtained, respectively, as follows:

$$\begin{aligned}&(I_x)_2 \frac{(c-b)w^3}{3}, \\ &(I_x)_3 = \frac{(d-c)w^3}{12}, \\ &(I_y)_2 = \frac{(c-b)^3 w}{3} + (c-b)b^2 w + (c-b)^2 bw, \\ &(I_y)_3 = \frac{(d-c)^3 w}{12} + \frac{(d-c)c^3 w}{2} + \frac{(d-c)^3 cw}{3},\end{aligned} \tag{8}$$

So, the (ROG) point of generalized trapezoidal fuzzy number can be calculated as q

$$r_x = \sqrt{\frac{(I_x)_1 + (I_x)_2 + (I_x)_3}{(((c-b)+(d-a)).w/2}},$$

$$r_y = \sqrt{\frac{(I_y)_1 + (I_y)_2 + (I_y)_3}{(((c-b)+(d-a)).w/2}}.$$

where the $(I_x)_1$, $(I_x)_2$, $(I_x)_3$, $(I_y)_1$, $(I_y)_2$, $(I_y)_3$ can be obtained from equations (7) and (8).

The Problem of ROG Method:

In this section, we first give a numerical example to present a problem of radius of gyration method (taken from (Wang and Lee, 2008). We assume that there are two triangular fuzzy numbers R_1, R_2, where R_1 = (1,2,3;1) and R_2 = (9, 10, 11;0.1).

Obviously, R^1 is smaller than R^2. So, by radius of gyration method, we can calculate,
$r_x(R_1) = 0.4082$, $r_y(R_1) = 2.0412$.

Therefore, $(r_x \times r_y)R_1 = 0.83333$.

Also we obtain $r_x(R_2) = 0.0408$, $r_y(R_2) = 10.0083$.

Therefore, $(r_x \times r_y)R_2 = 0.40858$.

According to the ROG method, we know that R_1 is bigger than R_2 as $r_{R1} > r_{R2}$. However, R_1 should be smaller than R_2 intuitively.

Now, we do this method for five fuzzy numbers which is taken from (Deng *et al.,* 2006)(See also in (Wang and Lee, 2008):

R_1 = (3,5,7;1), R_2 = (3,5,7;0.8), R_3 = (5,7,9,10;1), R_4 = (6,7,9,10;0.6), R_5 = (7,8,9,10;0.4). By radius of gyration method,

$r_1 = r_x \times r_y = 0.40824 \times 5.06622 = 2.06827$,
$r_2 = r_x \times r_y = 0.32659 \times 5.06622 = 1.65462$,
$r_3 = r_x \times r_y = 0.51176 \times 7.79346 = 3.98843$,
$r_4 = r_x \times r_y = 0.31622 \times 8.05191 = 2.54623$,
$r_5 = r_x \times r_y = 0.20000 \times 8.52447 = 1.70489$.

Therefore, we have

$r_2 < r_5 < r_1 < r_4 < r_3$.

According to the area between the radius and original point, this method is wrong. Intuitively, R_5 = (7,8,9,10;0.4) should be bigger than R_2 = (3,5,7;0.8), or R_4 = (6,7,9,10;0.6) maybebiggerthan R_1 = (3,5,7; 1). Now, we present a new method for ranking fuzzy numbers as follows:

(1)$r_y(A) > r_y(B) \Rightarrow A > B$.

(2)$r_y(A) < r_y(B) \Rightarrow A < B$.

$$(3) r_y(A) = r_y(B) \Rightarrow \begin{cases} r_x(A) > r_x(B) \Rightarrow A > B. \\ r_x(A) < r_x(B) \Rightarrow A < B. \\ r_x(A) = r_x(B) \Rightarrow A = B. \end{cases}$$

So, by using our method we can solve first example as following:

R_1 = (1,2,3;1), R_2 = (9, 10, 11; 0.1)

$\Rightarrow r_y(R_1) = 2.0412,\ r_y(R_2) = 10.0082,$
so, we have $R_1 < R_2$.

Now we are going to order the fuzzy numbers as given in the second example by our method as follows:

R_1 = (3,5,7;1), R_2 = (3,5,7;0.8), R_3 = (5,7,9,10;1), R_4 = (6,7,9,10;0.6), R_5 = (7,8, 9, 10; 0.4).

Our method concludes that:

$r_y(R_1) = 5.066228053,\ r_y(R_2) = 5.066228049,\ r_y(R_3) = 7.793464905,$
$r_y(R_4) = 8.051914887,\ r_y(R_5) = 8.524474567.$

Therefore,
$R_2 < R_1 < R_3 < R_4 < R_5$.

In another example, we want to order the following fuzzy numbers (taken from (Deng *et al.,* 2006)): R_1 = (0.4,0.5,1;1), R_2 = (0.4,0.7,1;1), R_3 = (0.4,0.9,1;1).

In the following tableau, we give the result of comparison of our method with three convenient methods for ordering fuzzy numbers.

method	Cheng	Chu-Tsao	ROG	our method
example	$R_1 < R_3 < R_2$	$R_1 < R_2 < R_3$	$R_1 < R_2 < R_3$	$R_1 < R_2 < R_3$

We saw our method ordered the fuzzy numbers R_1, R_2 and R_3 as well as ROG and Chu-Tsao methods. Also we saw the result of comparison of the mentioned fuzzy numbers by Cheng method is wrong (see in Fig 2).

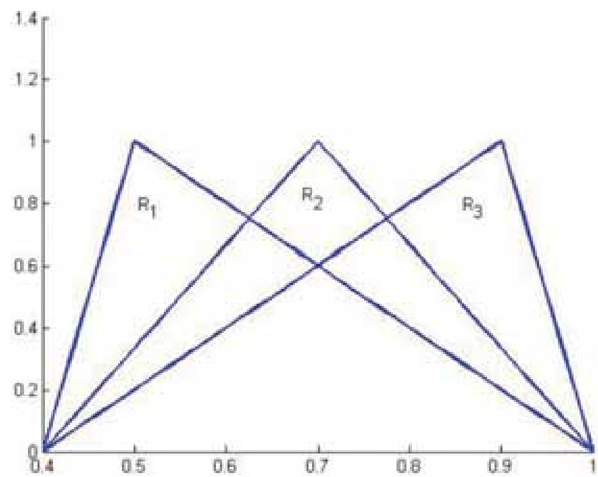

Fig. 2

Now consider the following fuzzy numbers:

R_1 = (1,3,4,6;0.6), R_2 = (1,3,4,6;0.9), R_3 = (5,9,12,15;0.2),

R_4= (5,9,12,15;0.7), R_5= (6,8,9,11;1).

The following tableau shows the result of our method and some convenient methods for ordering the above fuzzy numbers.

method	Cheng	Chu-Tsao	ROG	our method
example	$R_1 < R_2 < R_5 < R_3 < R_4$	$R_3 < R_1 < R_2 < R_4 < R_5$	$R_3 < R_1 < R_2 < R_4 < R_5$	$R_1 < R_2 < R_5 < R_3 < R_4$

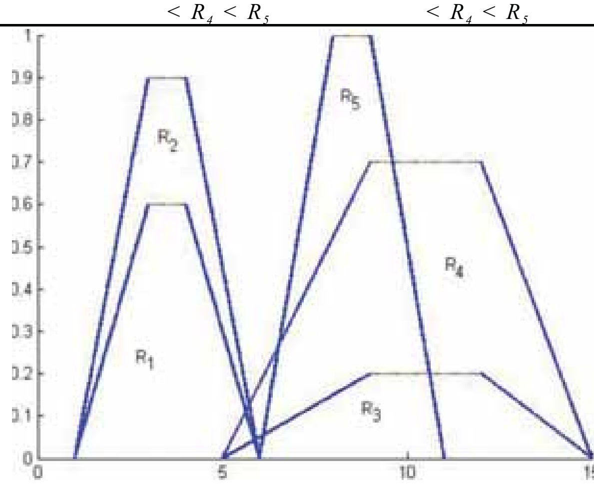

Fig. 3

As we saw our method obtained the correct result as well as Cheng method and also we found some shortcomings in ROG and Chu-Tsao methods. To sum up, our method is reasonable and effective for ranking fuzzy numbers according to the above examples.

Conclusion:

In this paper, we improved ROG method and presented a new method for ordering fuzzy numbers. By using this method, we solved some shortcoming of ROG method. For verifying efficiency of our method, we gave also used some comparative examples to illustrate the advantage of our method.

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