

## Crashworthiness Optimization of Foam-filled and Empty Spot-welded Columns

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**Abstract:** To improve crashworthiness efficiency, aluminum foam has been adopted as one of new filler materials in engineering. Introduction of the foam material changes the crash behavior of structural component and make necessary exploration of more sophisticated design optimization methodology. This paper presents a crashworthiness design of foam-filled and empty columns made from mild steels and joined by spot-weld. The numerical analysis is carried out by Abaqus software. Subsequently, the collapse behavior of spot-welded structures was experimentally characterized. Finally in order to find more efficient and lighter crush absorber and achieving minimum peak crushing force, response surface methodology (RSM) has been applied for optimizing the square foam-filled and empty spot-welded columns.

**Key word:** Foam, Spot-weld, RSM, Crashworthiness

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### INTRODUCTION

During recent years, the car body assembly techniques were dominated by spot-welding. Resistance spot-welding is a very quick, cheap and accessible technology to join metal sheets. Also it is controllable and it can be done automatically. Resistance spot-welding does not need special preparation of the parts before joining. On the other hand, weight-saving and impact safety requirements are calling for the application of light-weight materials and structures with high specific energy absorption to energy absorbing structures. Recently, much attention is given to the cellular material filled thin-walled structures. The studies showed that the interaction between metal or polymeric cellular material fillers and the supporting structures produces some desirable crushing behaviors and energy absorption properties. Among many optional cellular fillers, e.g., sawdust, honeycomb, polyurethane foam and metal foams, closed cell aluminum foam is the one gives some ideal performance. Peroni (2009) compared experimental results on the use of structural adhesives, laser-welding and spot-welding in structures subjected to crash. The obtained results demonstrate that continuously joined structures are at least equivalent to and generally better than spot-welded structures, and have further advantages typical of these joining solutions (higher stiffness and fatigue strength, improved vibration response, especially in the case of adhesive joints). Yujiang Xiang (2006) performed crashworthiness optimization of an empty spot-welded thin-walled hat section. Various spot-weld models were first used in a thin-walled hat section to compare with experimental works. An appropriate spot-weld model was then used in the transient nonlinear finite element analysis (FEA), and the number of spot-welds was selected as one of the design variables in optimization. The mass of the thin-walled tubes was optimized subject to constraints on the required mean crushing force and sectional stiffness. Hong-Wei Song (2005) investigated the interaction effect between aluminum foam and the metal spot-welded column. Based on their experimental examination, numerical simulation and analytical models, a systemic approach was developed to partition the energy absorption quantitatively into the foam filler component and the hat section component, and the relative contribution of each component to the overall interaction effect was therefore evaluated.

To seek for optimal crashworthiness of structure, some alternatives have been exploited for design optimization over the last decade. Response surface method (RSM) is a prevalent technique to model highly-nonlinear systems. The RSM was presented by Myers and Montgomery and advanced by other researchers (Myers, R.H., D.C. Montgomery, 2002; Oktem, H., T. Erzurumlu, 2005). The idea is to use some simple basis functions such as polynomials (Fang, H.B., K. Solanki, 2005; Forsberg, J., L. Nilsson, 2006) to approximate complex crashing response of a structure. This method has been employed to optimize many several other thin-

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walled structures with crashworthiness criterion (Yucheng Liu, 2008; Shujuan Hou, 2008; Shujuan Hou, 2009; Zarei, H.R., M. Kroger, 2008; Zarei, H.R., M. Kroger, 2008; Shujuan Hou, 2007).

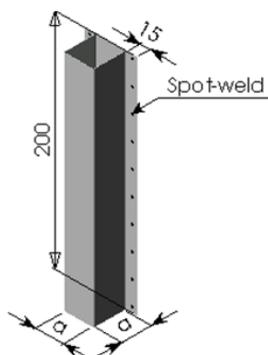
In this paper, the numerical quasi-static crushing responses of foam-filled spot-welded structures are investigated. The numerical crash analyse of foam-filled and empty tubes was performed using the ABAQUS finite element software and was validated by experimental results. To seek for the optimal crashworthiness design a set of designs are selected from the design space using the factorial design, which have different thickness column and side length (continues variables). And also the number of spot-welds (discrete variable) was selected as one of the design variables in optimization. For optimization of discrete variables, discrete optimization methods cannot be directly applied, because a large number of FE simulations would be required. In this paper thickness, side length of the column (continues variables) and also the number of spot-welds (discrete variable) are optimized by response surface method.

## **2- Numerical Analysis Using the Finite Element Method:**

The numerical simulations were carried out using the finite element software Abaqus/Explicit. In this simulation, a self-contact algorithm was used to prevent interpenetration during the folding of the columns and the spot-welds are modeled by surface-based tie constraints option in the Abaqus/Explicit. The plastic behavior of foam is taken into account using the CRUSHABLE FOAM and the CRUSHABLE FOAM HARDENING options in the Abaqus/Explicit software package (ABAQUS 6.7).

### **2-1 Geometry and Mechanical Properties of the Foam Filled Columns:**

The structures considered in this study are spot-welded thin-walled hat section, as illustrated in Fig. 1.



**Fig. 1:** Schematic drawings of the spot-welded column used in the current study

Design variables  $a$  and  $t$  are chosen as the design variables, and the constraints of these three design parameters are given as  $50 \leq a \leq 70$ ,  $1 \leq t \leq 2$  millimetre. The effects of these parameters on the following response of the filled spot-welded column evaluate for quasi-static loading. In this work, the lengths  $L$  of the spot-welded structures are a constant of 200 mm.

The spot-welded column material is mild steel with the following mechanical properties: Young's modulus  $E = 185$  GPa, yield stress  $\sigma_y = 220$  Mpa and ultimate stress  $\sigma_u = 260$  MPa. Furthermore, the value of Poisson ratio was assumed to be  $\nu = 0.3$ . This mechanical properties of this steel alloy were determined according to ASTM E8 standard, using the INSTRON 8802 servo hydraulic machine. The information for the aluminium foams with density  $220$  ( $\text{kg/m}^3$ ) have been reported in Ref (Ahmad, Z., D.P. Thambiratnam, 2009). The aluminum foam was modelled with the foam model of Dehspande and Fleck (2000) in Abaqus software.

### **2-2 Boundary Conditions and Element Formulation:**

For applying boundary conditions on the edges of the spot-welded columns, two rigid plates were used that were placed to the ends of the columns. All degrees of freedom in the lower plate and all degrees of freedom in the upper plate, except in the direction of longitudinal axis, were constrained. For this analysis, the linear element S4R, which is a four-node element, is suitable for analysis of thin shells, and element C3D8R, which is an 8-node linear brick element, is suitable for analysis of foam

**3 Problem Description:**

Response Surface Methodology (RSM) is a method for understanding the correlation between multiple input variables and one output variable. In this approach, an approximation  $Y$  to the response of the spot-welded columns is assumed a series of the basic functions in a form of,

$$Y = B_0 + \sum B_i X_i + \sum B_{ii} X_i^2 + \sum B_{ij} X_i X_j + \dots + \sum B_{ijkz} X_i X_j X_k X_z \tag{1}$$

in which  $Y$  is predicted response which is a dependent variable and  $B$  is regression coefficient.

The method of least-square can be used to determine the regression coefficient vector  $a$  by minimizing the errors  $E(a)$  between the FE analysis and the response function.

The regression coefficient vector  $B$  can be evaluate by  $\frac{\partial E(a)}{\partial x}$ , which is,

$$B = (\beta^T \beta)^{-1} (\beta^T y), \tag{2}$$

Where matrix  $\phi$  denotes the values of basis functions evaluated at these  $M$  sampling points, which is

$$B = \begin{bmatrix} \beta_1(x^{(1)}) & \dots & \beta_N(x^{(1)}) \\ \vdots & \ddots & \vdots \\ \beta_1(x^{(M)}) & \dots & \beta_N(x^{(M)}) \end{bmatrix} \tag{3}$$

substituting Eq. (2) into Eq. (1), the RS model can be fully defined. (Yucheng Liu, 2008; Shujuan Hou, 2008).

The crashworthiness of the spot-welded columns is expressed in terms of specific energy absorption SEA. The SEA is defined as

$$SEA = \frac{\text{Total energy absorption } E_{total}}{\text{Total structural weight}}$$

The optimization problem is;

$$\left\{ \begin{array}{l} \text{Minimize : } y = P(x) \\ \text{S.t } \quad SEA_x \geq SEA_{const.} \\ \quad \quad x^L \leq x \leq x^U \end{array} \right.$$

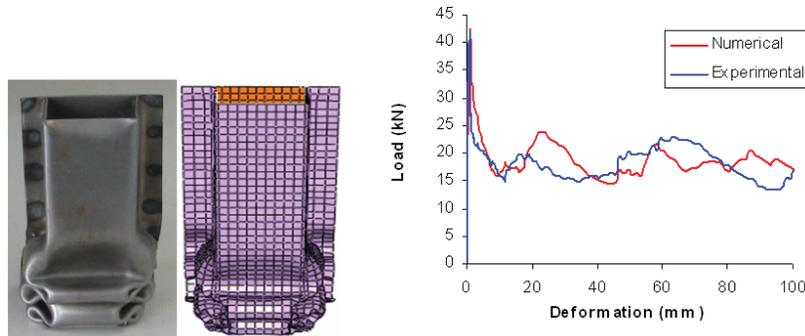
Where  $SEA_{const} = 10$  kJ/kg.  $x^L = (x_1^L, x_2^L, \dots, x_k^L)$  and  $x^U = (x_1^U, x_2^U, \dots, x_k^U)$  are respectively

the lower and upper bounds of the design variables. The numbers of spot-weld in foam-filled spot-welded columns are optimized by two methods. First method is to first optimize the geometry based on a section with a large number of spot-welds or a complete weld. Then the minimum number of spot-welds is determined by satisfying the optimization problem without changing the geometry obtained in the first step. The assumption here is that the crash behavior does not change much when the number of spot-welds is relatively large and the peak crashing force decreases with decreasing the number of spot-weld. This assumption is shown to be valid by the numerical results in Section 4-2. In second method, a large number of attempt would be to create RSM models corresponding to different numbers of spot-welds, and optimization results from all the RSM models are compared to determine the best solution. This method, could result in significant computational cost if the numbers of spot-welds to be considered is large.

**4 FE Models and Crashworthiness Analysis:**

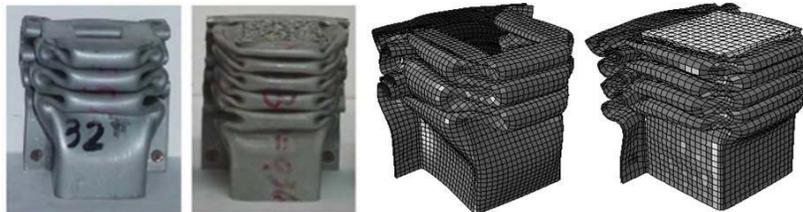
FE models are created for spot-welded columns and they are used for the crashworthiness analyses. For the two continuous variables (a, t) the factorial design method was adopted in design of experiments (DOE). FEA results of SEA and the maximum crushing force are acquired from the analyses and will later be used for constructing corresponding RS models.

Experimental tests were performed on some specimens in order to confirm the numerical results. For these tests, servo hydraulic INSTRON 8802 machine was used. Spot-welded columns, were joined by means of 6 mm diameter spots, positioned at the middle of the flanges width. The spot pitch was 20 mm. Also the load–displacement curves produced by numerical and experimental analyses are shown in Fig.2. A very good correlation between experiments and numerical simulations was observed.



**Fig. 2:** Comparison of the experimental and numerical results for the specimen with a=50 and t=1 mm

Comparing Fig.3 it can be deduced that the collapsed profiles of hat section have a stable manner while filled with aluminium foam, the structure has an even better stability. The collapse modes of the empty sections and their corresponding foam-filled sections are shown in Fig.3. If crushed in a progressive manner, foam-filled and empty columns share the similar collapse pattern, but the folding wavelength and the number of lobes may subject to change. In Fig.3, when each column was crushed 130 mm in the axial direction, the empty column formed 3 lobes, while the foam-filled column formed 5 lobes. Consequently, the folding wavelength reduced in the filled sections.



**Fig. 3:** Comparison of the experimental (Hong-Wei 2005) and numerical results for the empty and foam filled columns

The RS models are constructed based on the FEA results. In order to validate the set of design points and the orders of polynomials the different polynomial RS models are constructed, and then evaluated their accuracies. The results of approximations are summarized in Tables 1. Since the larger values of  $R^2$  and  $R^2_{adj}$  and the smaller values of RE and RMSE indicate a better fitness of the RS models, it is found that compared to other response functions the quartic polynomial functions provide the best approximation on the column's responses and therefore should be used for optimum design. As a result of the least square procedure, the quartic response functions of Max PL and SEA are, respectively, given as

$$SEA(h, w) = -58.84032 + 23.45144a + 40.8113t - 10.2706a^2 + 57.0207at - 151.1848a^2 + 0.9466a^3 - 0.7005a^2t - 38.4411at^2 + 120.3999a^3 - 0.01666a^4 - 0.2826a^3t + 2.2171a^2t^2 + 2.4426at^3 - 22.2399a^4 \quad (16)$$

$$\begin{aligned}
 &MAX PL(h, w) = \\
 &-70.5932 - 2.22358a + 314.3659t + 0.212204a^2 - 12.91394at - 39.2281t^2 - 0.0035853a^3 + 0.153390a^2t \\
 &+ 3.15175at^2 - 21.0336t^3 + 0.00002017a^4 - 0.00085954a^3t - .001031a^2t^2 - 0.66751at^3 + 9.5424a^4 \quad (17)
 \end{aligned}$$

and for the foam-filled column;

$$\begin{aligned}
 &SEA(h, w) = \\
 &145.2561 - 137.0743a + 165.8694t + 0.29.6592a^2 + 51.7643at - 273.6071a^2 - 2.8866a^3 - 7.2986a^2t \\
 &- 7.1715at^2 + 135.0399t^3 + 0.1026a^4 + 0.4160a^3t + 0.0130a^2t^2 + 0.0150at^3 - 24.0639t^4 \quad (18)
 \end{aligned}$$

$$\begin{aligned}
 &MAX PL(h, w) = \\
 &455.6046 - 22.0220a - 364.8464t + 0.4408a^2 + 10.5192at + 192.02506t^2 - 0.0041a^3 - 0.1083a^2t \\
 &- 2.3375at^2 - 54.2698t^3 + 0.000014a^4 + 0.00045a^3t + 0.01016a^2t^2 + 0.2250at^3 - 7.1381t^4 \quad (19)
 \end{aligned}$$

**Table 1:** Accuracy of different polynomial RS models for spot welded columns

RS model	R <sup>2</sup>	R <sup>2</sup> <sub>adj</sub>	RMSE	RE interval (%)
Quadratic polynomial	0.9990	0.997	0.0099	[-1.8, 2.1]
Cubic polynomial	0.9994	0.998	0.0093	[-0.7, 1.5]
Quartic polynomial	0.9999	0.999	0.0012	[-0.4, 1.0]

The optimal results acquire using the nonlinear programming (fmincon), which is provided by MATLAB. “fmincon” attempts to find a constrained minimum of a scalar function of several variables starting at an initial estimate (MATLAB User’s Manual,). Table 2 shows the optimized tube geometry and its energy absorption and peak crushing force.

Comparing table.2 it can be deduced that using of foam filled column results lesser peak crushing force relative to empty column in same SEA that it is desirable.

**Table 2:** Optimal square hat section designs

Spot-welded column	Optimal design variables (cm)	SEA (kJ/kg)		P <sub>m</sub> (kN)	
		RSM	FEM	RSM	FEM
Optimum foam-filled column	a=50, t=1.52mm	10	9.9	70	69.5
Optimum empty column	a=50, t=1.81mm	10	10.0	85	87

Fig. 4 shows the RS of absorbed energy and peak force as a function of  $\alpha$  and  $t$  at a constant deflection of 120 mm. It can be seen that in Fig. 4 with increasing  $t$  and decreasing  $\alpha$ , the SEA increases and with increasing  $t$  and  $a$  the peak force increases.

**4-1 Step 2: Number of Spot-weld Optimization:**

In this step, the optimized variables in Table 2 were selected and the first algorithm optimized the number of spot-welds. The SEA for various  $n$  ( $n=3, 4 \dots$ ) were obtained from FEA, and the smallest  $n$  with a specific energy absorption greater than 10 kJ was selected as the final solution. Fig. 5 shows the trend of SEA and peak crushing force with changes of  $n$ . Fig. 5 show that the SEA for ( $n < 10$ ) do not meet the constraint of ( $SEA \geq 10$ ); therefore, the minimum number of spot-welds is 10. The peak crushing force corresponding to the optimum solution is lower than the constraint, that it is desirable.

Fig. 5 also shows that the trend of the SEA tends to be flat from  $n = 10$  to a complete weld. This verifies the assumption of the first optimization algorithm that the number of spot-welds can be decoupled from the rest of design variables for the crushing force when the number of spot-welds is large enough. The fluctuation of the values of SEA for ( $n > 10$ ) is due to the highly nonlinear behaviors of crushing processes, because varying the number of spot-welds may result in different deformation modes and shapes (Oktem, H., T. Erzurumlu, 2005).

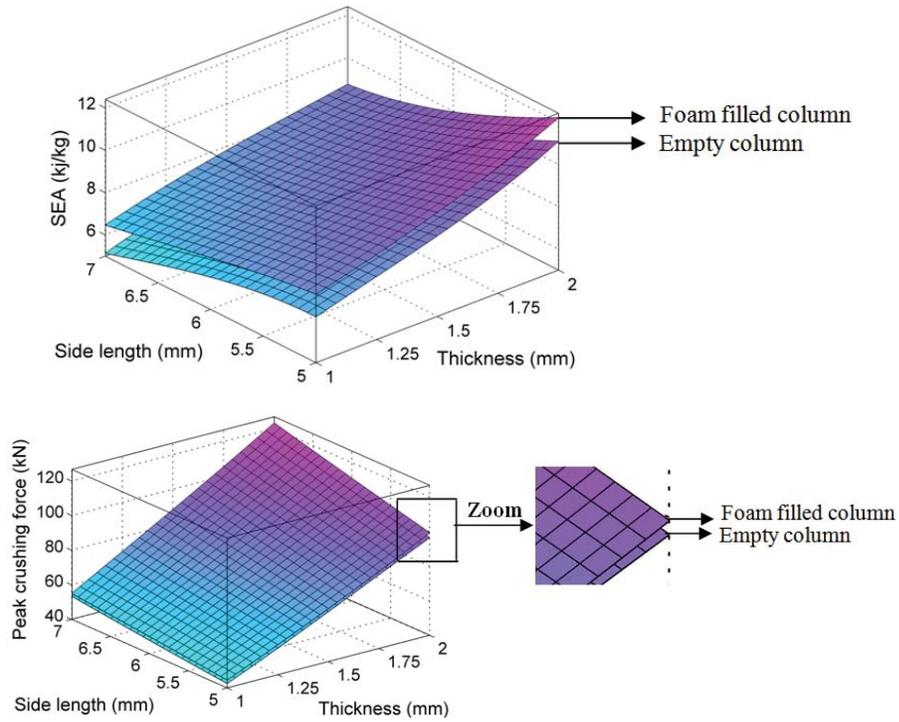


Fig. 4: Response surface of SEA and Peak force for the spot-welded columns

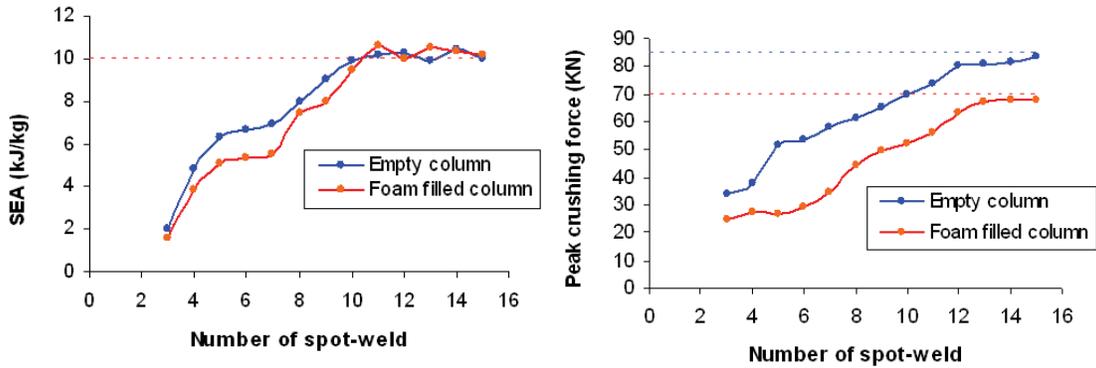


Fig. 5: Variation of the specific energy absorption and peak crushing force vs. number of spot-welds.

4.2. Second Optimization Algorithm:

To verify the optimal result obtained by the first algorithm, the second algorithm was also used to compute the optimum design variable. This is done by enumerating the number of spot-welds  $n$  and optimizing the design variables for each  $n$ . The values of SEA based on the RSM are given in Table 3.

Table 3: Comparison of the two optimization algorithms

number of spot-weld	t (mm)		A (mm)		P <sub>max</sub> (kN)		SEA (kJ/kg)	
	Foam filled	empty	Foam filled	empty	Foam filled	empty	Foam filled	empty
8	-	-	-	-	-	-	-	-
10	1.54	1.83	50	50	69.5	86	10.0	10.0
12	1.49	1.78	50	50	70.2	85.5	10.0	10.0
14	1.52	1.8	50	50	71.0	86.5	10.0	10.0

It can be seen from Tables 3 that there are no meaningful changes for design variables (a, t), constraints (SEA) and Peak crushing force for ( $n \geq 10$ ) and also it can be seen for ( $n \leq 10$ ) the SEA is lesser than 10(kJ/kg) and do not satisfy the optimization problem. Table 3 also verifies that the number of spot-welds can be separated from the other design variables for the optimization. The final optimal results of the first and second algorithms are very close, as given in Table 5. However, the first algorithm is better than the second algorithm, because that requires less FE simulations. For example, if the number of spot-welds  $n$  varies from 8 to 13, the required number of FEA in foam filled columns is 31 in the second algorithm (25 for factorial design method plus 6 for enumerations), while it is 100 in the second algorithm (4 enumerations each with 25 for factorial design method).

**Table 3:** Comparison of the two optimization algorithms

number of spot-weld	t (mm)		A (mm)		P <sub>max</sub> (kN)		SEA (kJ/kg)	
	Foam filled	empty	Foam filled	empty	Foam filled	empty	Foam filled	empty
First algorithm	1.52	1.81	50	50	70	85	10.0	10.0
second algorithm	1.54	1.83	50	50	69.5	86	10.0	10.0

**Conclusions:**

In this paper, crashworthiness optimization of spot-welded columns was studied when no spot-weld failure occurred. The number of spot-welds and cross-sectional of spot-welded columns are optimized. Two different design algorithm were used for optimization in this study. The effect of side length and wall thicknesses on SEAs and peak crushing forces are also presented through the plots of the response surfaces, which show with increasing t and decreasing a, the SEA increases and with increasing t and a the peak force increases. The illustrative example indicated that for optimization of number of spot welds, the first algorithm is more efficiency than the second algorithm.

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