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Effects of Stenosis on Non-newtonian Flow of Blood in Blood Vessels

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Abstract: An attempt has been made to investigate the wall shear stress, resistance parameter and flow rate across mild stenosis situated symmetrically on steady blood flow through blood vessels with uniform or non-uniform cross-section by assuming the blood to be Non-Newtonian, incompressible and homogeneous fluid. An analytic solution for Power law Fluid, Bingham Plastic fluid, Casson Fluid has been obtained. For the physiological insight of the problem various parameters of systemic and pulmonary artery are taken and the study reveals that as the height of the stenosis increases in blood vessels, the shear stress and resistance parameter steadily increases, whereas, flow rate decreases steadily.

Keywords: Power Law Fluid, Casson Fluid, Bingham Plastic fluid, Blood flow, Arterial stenosis, pulmonary artery; systemic artery; wall shear stress, resistance parameter, flow rate.

INTRODUCTION

Diseases in the blood vessels and in the heart, such as heart attack and stroke, are the major causes of mortality world wide. The underlying cause for these events is the formation of lesions, known as atherosclerosis, in the large and medium –sized arteries in the human circulation. These lesions and plaques can grow and occlude the artery and hence prevent blood supply to the distal bed. Plaques with calcium in them can also rupture and initiate the formation of blood clots (thrombus). The clots can form as emboli and occlude the smaller vessels that can also result in interruption of blood supply to distal bed. Plaques formed in coronary arteries can lead to heart attacks and clots in the cerebral circulation can result in the stroke. There are a number of risk factors for the presence of atherosclerotic lesions. The common sites for the formation and development of atherosclerosis include the coronary arteries, the branching of the subclavian and common carotids in the aortic arch, the bifurcation of the common carotid to internal and external carotids especially in the carotid sinus region distal to the bifurcation, the renal arterial branching in the descending aorta and in the ileo-femoral bifurcations of the descending aorta. The common feature in the location for the development of the lesion is the presence of curvature, branching, and bifurcation present in these sites. The fluid dynamics at these sites can be anticipated to be vastly different from other segments of the arteries that are relatively straight and devoid of any branching segments. Hence, several investigators have attempted to link the fluid dynamically induced stress with the formation of atherosclerotic lesion in the human circulation (A.C. Guyton and J.E. Hall, 2006).

By assuming the artery to be circularly cylindrical in shape, (B.K. Mishra 2003) discussed characteristics of blood in stenosed artery and the stenosis to be symmetric about the axis of artery. (B.K. Mishra and T.C. Panda 2005) studied the flow of blood in stenosed artery for a power law fluid and Casson fluid. Chaturani (Chaturani P. and R.P. Sany 1985) investigated the various aspects of blood flow in stenosed artery assuming the blood to be Non – Newtonian. (Young D.F. and Tsai F.Y. 1973) discussed the various characteristics of steady and unsteady flow of blood in stented arteries.

The initiation and development of atherosclerotic plaques is depicted in Figure 1 and 2 (www.alegent.com/18576.cfm, www.vhn.ca/patient_menu.php). The blood vessels in figure 2 that we are talking about are the arteries. They are the structures that carry blood from the heart to all the organs and tissues of the body including brain, kidneys, gut, muscles, and the heart itself. Below are a series of illustrations that will help us to understand the process of atherosclerosis (vascular disease) and the kinds of problems that can arise

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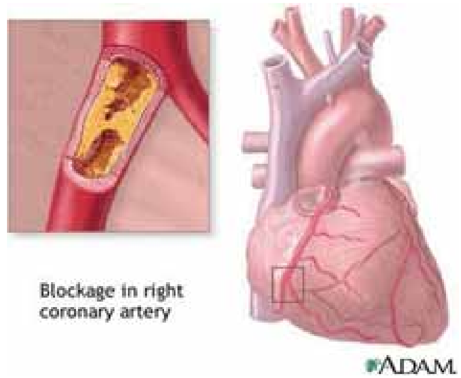

Fig. 1: Blocked in the right coronary artery [7]

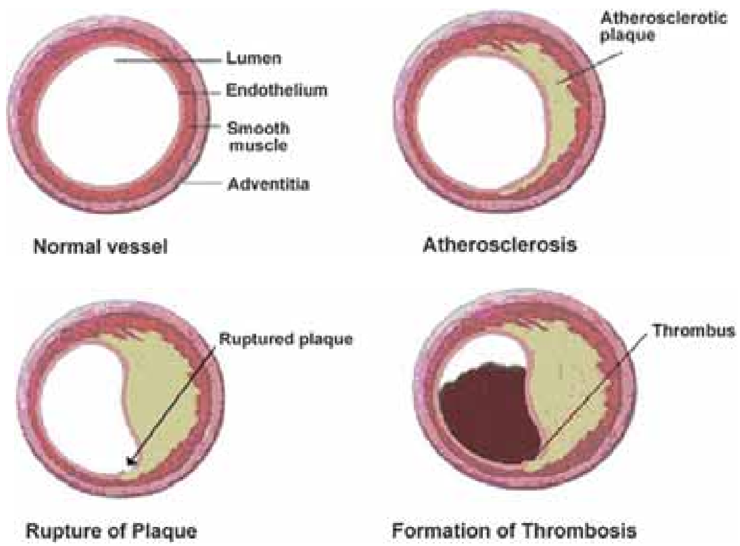

Fig. 2: Development of atherosclerotic plaques [8]

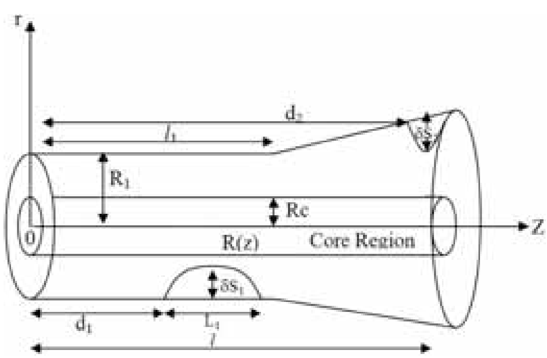

Fig. 3: Physical model and coordinate system.

in this condition. In Fig.2 (a), we are looking at the vessel sliced across. This vessel is normal - like we all come equipped with at birth. The white space marked "lumen" is where the blood from the heart would flow to the rest of our body. Fig. 2(b) is showing the early stages of vascular disease with the formation of what is called a "plaque" generally made up of cholesterol and sometimes calcium. Fig. 2(c) shows a very serious problem. When we observe the six o'clock position (tip of the arrow), we see that the smooth layer or sleeve - called the endothelium - has ruptured. Up until now that layer has prevented any direct contact between the blood flowing by and the material within the plaque. Once the material underneath the sleeve is exposed to the passing blood, the blood begins to clot. In this process, a narrowing that might have been 40 or 50% of the cross sectional area of the artery can become an 80, 90, or l00% narrowing within seconds or minutes. This is the process that causes most heart attacks and sudden death (Fig. 2(d)).

In the present paper, blood is assumed to be Non-Newtonian, incompressible and homogeneous fluid; cylindrical polar co-ordinate is used, with the axis of symmetry of artery taken as Z-axis. The stenoses are mild and the motion of the fluid is laminar and steady. The inertia term is neglected, as the motion is slow. No body force acts on the fluid and there is no slip at the wall.

Nomenclature:

- ρ = Density of Blood
- μ = Viscosity of Blood
- P = Pressure
- R_1 = Radius of uniform portion of tube
- R (z) = Radius of obstructed portion of tube
- $R_{sn}(z)$ = Radius of obstructed portion due to the nth stenosis of tube
- δS_n = Amplitude of nth stenosis.
- L_n = Length of nth stenosis
- d_n = Location of nth stenosis
- l_1 = Length of uniform portion of tube
- l = length of tube
- K = Wall exponent parameter
- τ = Wall shear stress
- τ_0 = measure of yield stress
- e= strain rate=- $\left(\frac{du}{dr}\right)$
- u=velocity of fluid
- R_c = Radius of the core region of the tube
- λ= Resistance to flow at the wall for the flow of blood
- λ_0= Resistance to flow at the wall for the flow of blood in uniform portion of tube
- λ` = Resistance parameter

We assume the following non-dimensional quantities:

$$Z' = (Z/l), d_n' = (d_n/l), L_n' = (L_n/l), l_1' = (l_1/l), \delta S_n' = (\delta S_n/R_1), R'(z) = (R(z)/R_1), R_1' = R_1/l$$
$$\lambda' = \frac{\lambda}{\lambda_0}$$

Basic Equations:

In the present analysis, it is assumed that that the stenosis develops in the arterial wall in an axially symmetric manner and depends upon the axial distance z, and the height of its growth (Figure 3).In such a case the radius of the artery, R(z), can be written as follows:

$$R(Z) = R_1(z) = R_1; 0 \le z \le d_1 \,\&\, d_1 + L_1 \le z \le l_1$$
$$R(z) = R_{s1}(z) = R_1 - \delta S_1/2.\left[1 + \cos\left(\frac{2\pi}{L_1}\right)(z - d_1 - L_1/2)\right]; d_1 \le z \le d_1 + L_1 \tag{1}$$
$$R(z) = R_2(z) = R_2(z); l_1 \le z \le d_2 \,\&\, d_2 + L_2 \le z \le l$$
$$R(z) = R_{s2}(z) = R_2(z) - \delta S_2/2.\left[1 + \cos(2\pi/L_2)(z - d_2 - L_2/2)\right]; d_2 \le z \le d_2 + L_2$$

Effect of Wall Shear Stress:

Power Law Model:

Table 1: Variation of τ against $\delta S`_1$ for K= -0.001, 0, 0.001.

$\delta S`_1$	τ		
	K= 0.001	K=0	K=-0.001
.027	35.324	20.174	5.024
.034	38.354	23.20	8.050
.040	41.384	26.234	11.084
.046	44.414	29.264	14.114
.053	47.444	32.294	17.144
.060	50.474	35.324	20.174

Bingham Plastic Model:

Table 2: Variation of τ against $\delta S`_1$ for K= -0.001, 0, 0.001

$\delta S`_1$	τ		
	K= 0.001	K=0	K=-0.001
.027	33.65	33.603	33.556
.034	37.71	37.663	37.616
.040	41.77	41.683	41.636
.046	45.83	45.723	45.636
.053	49.89	49.763	49.716
.060	53..92	53.905	53.858

Casson Model:

Table 3: Variation of τ against $\delta S`_1$ for K= -0.001, 0, 0.001

$\delta S`_1$	τ		
	K= 0.001	K=0	K=-0.001
.027	57.89	50.120	42.35
.034	57.8956	50.127	42.359
.040	57.900	50.132	42.364
.046	57.9053	50.137	42.369
.053	57.910	50.143	42.374
.060	57.916	50.148	42.379

Effect of Flow Rate:

Power Law Model:

Table 4: Variation of Q against $\delta S`_1$ for K= -0.001, 0, 0.001

$\delta S`_1$	Q		
	K= 0.001	K=0	K=-0.001
.027	40.48	39.89	39.24
.034	40.41	39.77	39.21
.040	40.37	39.74	39.17
.046	40.33	39.72	39.12
.053	40.30	39.69	39.07
.060	40.25	39.64	39.04

Bingham Plastic Model:

Table 5: Variation of Q against $\delta S`_1$ for K= -0.001, 0, 0.001

$\delta S`_1$	Q		
	K= 0.001	K=0	K=-0.001
.027	75.21	74.25	73.29
.034	75.18	74.22	73.26
.040	75.15	74.19	73.23
.046	75.12	74.16	73.20
.053	75.09	74.13	73.17
.060	75.06	74.10	73.14

We assume one stenosis each in uniform and non-uniform portion of the artery (Figure 3).

To observe explicitly the effect of various parameters on resistance, wall shear stress and viscosity to the flow, the following function has been assumed for the artery radius, which is non-uniform.

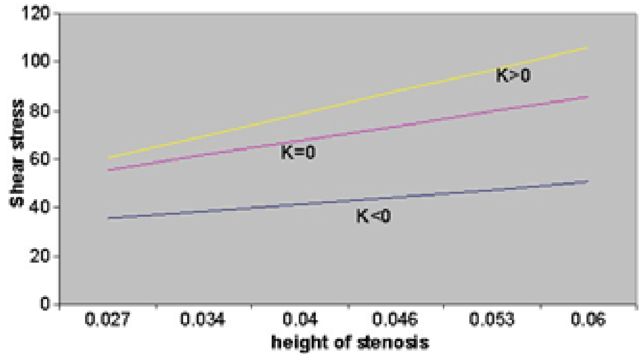

Fig. 4: Variation of τ against $\delta S`_1$ for various value of K

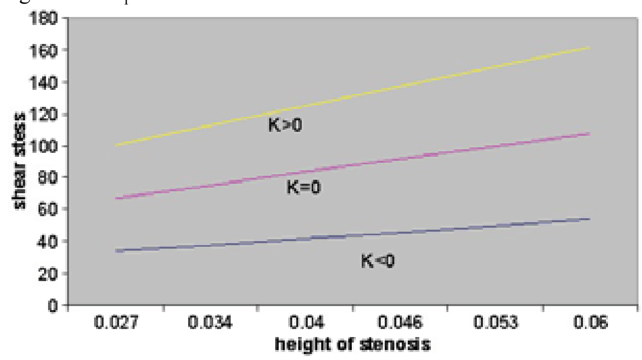

Fig. 5: Variation of τ against $\delta S`_1$ for K= -0.001, 0, 0.001

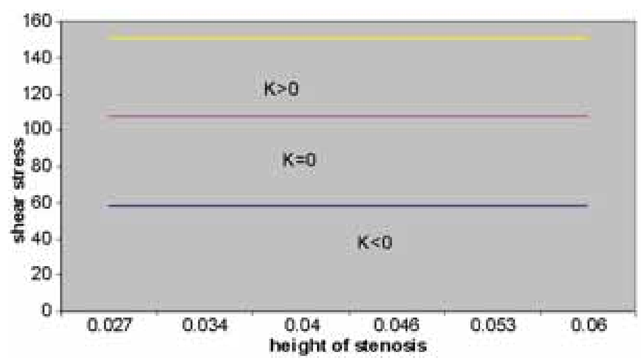

Fig. 6: Variation of τ against $\delta S`_1$ for K= -0.001, 0, 0.001

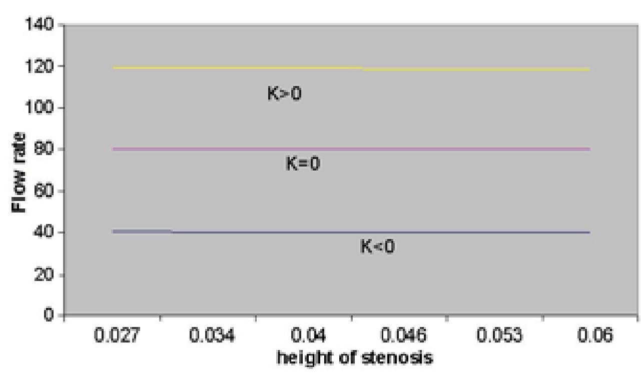

Fig. 7: Variation of Q against $\delta S`_1$ for various value of K

$$R(z) = R_1 e^{K(z-l_1)^2}; l_1 \leq z \leq l \tag{2}$$

For the steady flow through circular artery, the wall shear stress is given by,

$$\tau = \frac{r\frac{dp}{dz}}{2} = \frac{rG}{2} \tag{3}$$

Where, $G = \left(\frac{dp}{dz}\right)$ is the pressure gradient (4)

The flow rate Q through the artery, is the sum of the flow through the core region and that in the peripheral region, i.e.,

$$Q = Q_{core} + Q_{peripheral} \tag{5}$$

where the flow rate through the core and peripheral region respectively is given by

$$Q_{core} = u_c \pi R_c^2 \tag{6}$$

$$Q_{peripheral} = \int_{R_c}^{R} 2\pi u r dr \tag{7}$$

The resistance to flow at the wall for the flow of blood can also be expressed as

$$\lambda = \frac{dp}{Q}$$

Development of the Model:

Case 1: Power Law Model:

The constitutive relationship for the power fluid is given by the relationship

$$\tau = \mu e^{n} \qquad (n<1) \tag{8}$$

The velocity of the fluid through the tube thus can be expressed in terms of

$$u = -\left(\frac{G}{2\mu}\right)^{1/n}\left(\frac{nr^{n+1/n}}{n+1}\right) + C \tag{9}$$

Where C is constant of integration

Applying the boundary condition: u=0; r=R, we have,

$$C = -\left(\frac{G}{2\mu}\right)^{1/n}\left(\frac{nR^{n+1/n}}{n+1}\right) \tag{10}$$

Thus the velocity of the fluid in the tube is given by equation (11)

$$u = \left(\frac{G}{2\mu}\right)^{1/n}\left(\frac{n}{n+1}\right)\left[R^{n+1/n} - r^{n+1/n}\right] \tag{11}$$

The flow through the artery can be obtained from the basic equation

$$Q = \int_0^R 2\pi u r dr \tag{12}$$

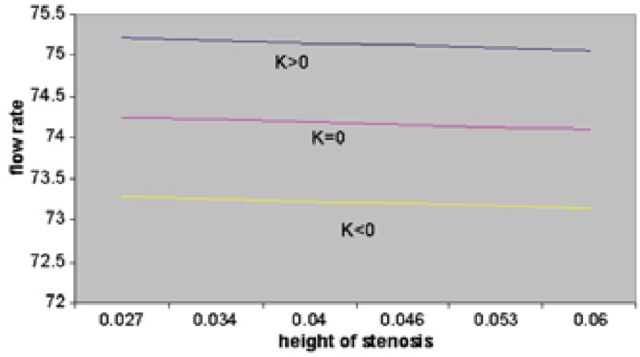

Fig. 8: Variation of Q against $\delta S`_1$ for K= -0.001, 0, 0.001

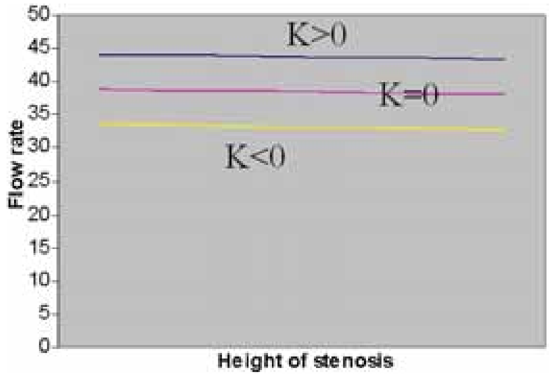

Fig. 9: Variation of Q against $\delta S`_1$ for K= -0.001, 0, 0.001

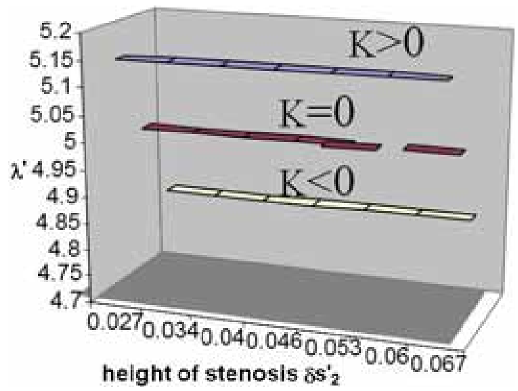

Fig. 10: Variation of $\lambda`$ against $\delta S`_2$ for various value of K

For n = ½, we get the expression of flow through the blood vessel as

$$Q = \frac{\pi P^2}{20\mu^2} R_1^5 \left[l + \left\{ 5Kl_1^2 \right\} \left\{ l - l_1 \right\} - 5 \left\{ \delta S_1' L_1 + \delta S_2' L_2 \right\} \right] \tag{13}$$

Also the expression of wall shear stress through the blood vessel is given in equation (14)

$$\tau = R_1^{-3/2} \left[l + \left\{ \frac{3Kl_1^2}{2} \right\} \left\{ l - l_1 \right\} + 1.5 \left\{ \delta S_1' L_1 \left\{ 1 + \frac{\pi^2}{L_1^2} (d_1 + L_1/2) \right\} + \delta S_2' L_2 \left\{ 1 + \frac{\pi^2}{L_2^2} (d_2 + L_2/2) \right\} \right\} \right] \tag{14}$$

Casson Model:

Table 6: Variation of Q against δS`$_1$ for K= -0.001, 0, 0.001

δS`$_1$	Q		
	K= 0.001	K=0	K=-0.001
.027	44.19	38.88	33.57
.034	44.04	38.73	33.42
.040	43.86	38.55	33.24
.046	43.69	38.38	33.07
.053	43.53	38.22	32.91
.060	43.36	38.05	32.74

Effect of Resistance Parameter:

Power Law Model:

Table 7: Variation of λ` against δS`$_2$ for K= -0.001, 0, 0.001

δS`$_2$	λ`		
	K= -0.001	K=0	K=0.001
.027	5.150996	5.0028200003	4.854644
.034	5.151416	5.0032400003	4.855064
.040	5.151776003	5.003600003	4.855424003
.046	5.152136	5.0039600003	4.855784
.053	5.152556003	5.0043800003	4.856204003
.060	5.152976003	5.0048000003	4.856624003
.067	5.153126001	5.005200003	4.856927003

Bingham Plastic Model:

Table 8: Variation of λ` against δS`$_2$ for K= -0.001, 0, 0.001

δS`$_2$	λ`		
	K= -0.001	K=0	K=0.001
.027	41.78299	41.000855	42.55743
.034	41.7856273	41.006193	42.56849132
.040	41.78706878	41.01262878	42.56150878
.046	41.7892678	41.01452678	42.56260128
.053	41.79114558	41.01670558	42.56558588
.060	41.7936426	41.02060523	42.5685743

Table 9: Variation of λ` against δS`$_2$ for K= -0.001, 0, 0.001

δS`$_2$	λ`		
	K= -0.001	K=0	K=0.001
.027	32.87880086	32.105365	31.33192914
.034	32.88328086	32.109845	31.33640914
.040	32.88550086	32.111765	31.33822914
.046	32.88712086	32.113685	31.34024914
.053	32.88936068	32.115925	31.34248914
.060	32.89160086	32.118165	31.344472914
.067	32.89384086	32.120405	31.34696914

Table 10: Variation of λ` against δS`$_1$ and δS`$_2$ for K= -0.001, 0, 0.001

λ`	δS`$_1$	δS`$_2$		
		K= -0.001	K=0	K=0.001
.027	.027	42.55967298	41.78523198	41.01079198
.034	.034	42.57612321	41.7872743	41.0136148
.040	.040	42.5796876	41.79346876	41.01902876
.046	.046	42.5699123	41.7954198	41.02301728
.053	.053	42.57614558	41.80170558	41.02726558
.060	.060	42.557917243	41.80370216	41.03316458

For n =1/3, Resistance to flow at the wall for the flow of blood is given by

$$\lambda = 48\frac{\mu^3}{\pi}\int_0^l \left[2R^4 - 2R_c^4\right]^{-1} dz$$

The resistance to flow at the wall for the flow of blood in uniform portion of blood vessel is

$$\lambda_0 = 48\frac{\mu^3}{\pi}\int_0^l \left[2R_1^4 - 2R_c^4\right]^{-1} dz$$

Thus the resistance parameter for the flow of blood in the blood vessel is expressed as

$$\lambda' = \left(l - L_1\right) + L_1\left[1 + 6\delta S'_1 + R_c^6\right] + L'_2\left[1 + 6\delta S'_2 + R_c^6\right] + \left(1 + R^6{}_c\right)\left(l - l'_1 - L_2\right) - 2K\left(l - l'_1\right)^3 \tag{15}$$

Case 2: Bingham Plastic Model:

The constitutive relationship for the Bingham Plastic fluid is given by the relationship

$$\tau = \tau_0 + \mu e \tag{16}$$

The velocity of the fluid thus can be expressed in terms of

$$u = -\frac{1}{\mu}\left(\frac{r^2 G}{4}\right) + \frac{\tau_0 r}{\mu} + C \tag{17}$$

where C is constant of integration

Applying the boundary condition: u=0; r=R, we have

$$C = \frac{1}{\mu}\left(\frac{R^2 G}{4}\right) - \frac{\tau_0 R}{\mu} \tag{18}$$

Thus we have the expression of velocity of the fluid given in equation (19)

$$u = \left(\frac{G}{\mu}\right)\left[\frac{\left(R^2 - r^2\right)}{4} - \frac{R_c\left(R - r\right)}{2}\right] \tag{19}$$

Where R_c is radius of core region, and $\tau_0 = \dfrac{R_c G}{2}$ (20)

The velocity in the core region is thus given by

$$u_c = \frac{1}{\mu}\left(\frac{G}{4}\right)\left(R^2 - R_c^2\right) - \frac{\tau_0\left(R - R_c\right)}{\mu} \tag{21}$$

The expression of flow in the core region can thus be expressed as

$$Q_{core} = u_c \pi R_c^2 = \frac{G\pi}{\mu}\left[\frac{\left(R^2 - R_c^2\right)R_c^2}{4} - \frac{R_c^2}{2}\left(R - R_c\right)\right] \tag{22}$$

Also the flow through the peripheral region can be expressed as

$$Q_{peripheral} = \left(\frac{\pi G}{\mu}\right)\left[\frac{R^4}{8} - \frac{R_c^2 R^2}{4} - \frac{5}{24}R_c^4 + \frac{R_c^3 R}{2} - \frac{R_c R^3}{6}\right] \tag{23}$$

Substituting the value of Q_{core} and $Q_{peripheral}$ in equation (5), we get the expression for the flow through the blood vessel as

$$Q = \left(\frac{\pi G}{\mu}\right)\int_0^1 \left[\frac{R^4}{8} + \frac{R_c^4}{24} - \frac{R_c R^3}{6}\right] dz \quad (24)$$

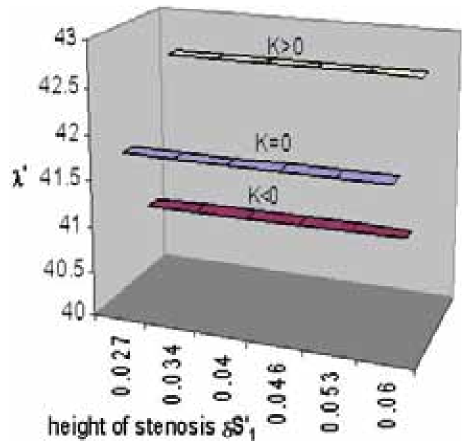

Fig. 11: Variation of λ` against $\delta S`_1$ for K= -0.001, 0, 0.001

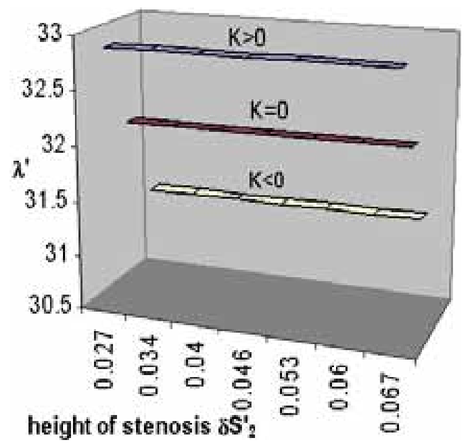

Fig. 12: Variation of λ` against $\delta S`_2$ for K= -0.001, 0, 0.001

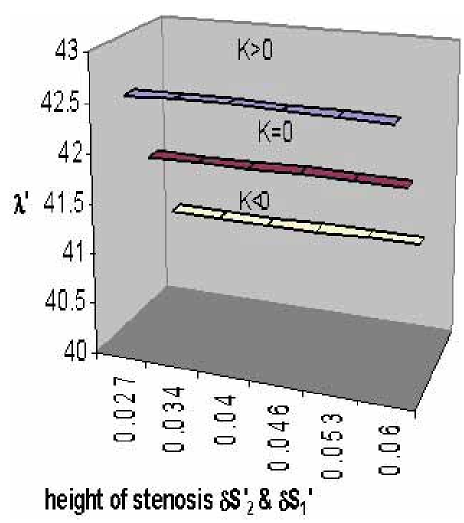

Fig. 13: Variation of λ` against $\delta S`_1$ and $\delta S`_2$ for K= -0.001, 0, 0.001

We get the expression of flow through the artery as

$$Q=\frac{1}{24}\left[\left\{3R_1^4+R_c^4-4R_cR_1^3\right\}l+\left\{R_1^4-R_cR_1^3\right\}\left\{12Kl_1^2\right\}\left\{l-l_1\right\}-12\left\{R_1^4-R_cR_1^3\right\}\left\{\delta S_1'L_1+\delta S_2'L_2\right\}\right] \quad (25)$$

The expression of wall shear stress can be expressed as given in equation (26)

$$\tau=\left[\begin{array}{l}\left\{R_1-\frac{3R_1^5}{R_c^4}+\frac{4R_1^4}{R_c^3}\right\}l+\left\{R_1-\frac{15R_1^5}{R_c^4}+\frac{16R_1^4}{R_c^3}\right\}\left\{Kl_1^2\right\}\left\{l-l_1\right\}+\\ \left\{\delta S_1'L_1\left\{1+\frac{\pi^2}{L_1^2}\left(d_1+L_1/2\right)\right\}+\delta S_2'L_2\left\{1+\frac{\pi^2}{L_2^2}\left(d_2+L_2/2\right)\right\}\right\}\left\{\frac{15R_1^4}{R_c^4}-\frac{16R_1^3}{R_c^3}\right\}\end{array}\right] \quad (26)$$

The resistance to flow at the wall for the flow of blood is given by

$$\lambda=\frac{dp}{Q}=\frac{\mu}{\pi}\int_0^l\left[\frac{R^4}{8}+\frac{R_c^4}{24}-\frac{R_cR^3}{6}\right]^{-1}dz$$

The resistance to flow at the wall for the flow of blood in uniform portion of blood vessel

$$\lambda_0=\frac{\mu}{\pi}\int_0^l\left[\frac{R_1^4}{8}+\frac{R_c^4}{24}-\frac{R_cR_1^3}{6}\right]^{-1}dz$$

Thus the resistance parameter for the flow of blood in the blood vessel is expressed as

$$\lambda'=8L_1'\left[1+4\delta S_1'-\frac{R_c^4}{3}+\frac{4R_c}{3}-4R_c\delta S_1'\right]+\left(l-L_1'\right)+$$

$$8\left[\left(1+\frac{4R_c}{3}-\frac{R_c^4}{3}\right)\left(d_2'-l_1'\right)-\frac{4K\left(1-R_c\right)}{3}\left(d_2'-l_1'\right)^3\right]+$$

$$8L_2'\left[1+4\delta S_2'-\frac{R_c^4}{3}+\frac{4R_c}{3}\right]+8\left[-\frac{4K\left(1-R_c\right)}{3}\left\{\left(d_2'+L_2'-l_1'\right)^3-\left(d_2'-l_1'\right)^3\right\}\right]$$

$$+8\left[\left(1+\frac{4R_c}{3}-\frac{R_c^4}{3}\right)\left(l-d_2'-L_2'\right)-\frac{4K\left(1-R_c\right)}{3}\left\{\left(l-l_1'\right)^3-\left(d_2'+L_2'-l_1'\right)^3\right\}\right] \quad (27)$$

Case III: Casson Model:

The constitutive relationship for the Casson fluid is given by the relationship

$$\tau^{1/2}=\tau_0^{1/2}+\left(\mu e\right)^{1/2} \quad (28)$$

The velocity of the fluid thus can be expressed in terms of

$$u = -\frac{1}{\mu}\left(\frac{r^2 G}{4}\right) - \frac{\tau_0 r}{\mu} + \frac{2}{\mu}\left(\frac{\tau_0 G}{2}\right)^{1/2}\left(\frac{2r^{3/2}}{3}\right) + C \tag{29}$$

where C is constant of integration.
Applying the boundary condition: u=0; r=R,

$$C = \frac{1}{\mu}\left(\frac{R^2 G}{4}\right) + \frac{\tau_0 R}{\mu} - \frac{2}{\mu}\left(\frac{\tau_0 G}{2}\right)^{1/2}\left(\frac{2R^{3/2}}{3}\right)$$

For $r>R_c$, the expression for the velocity profile is

$$u = \frac{1}{\mu}\left(\frac{G}{4}\right)\left(R^2 - r^2 + 2R_c R - 2R_c r - \frac{8}{3}R_c^{1/2}R^{3/2} + \frac{8}{3}R_c^{1/2}r^{3/2}\right) \tag{30}$$

The core velocity is given by

$$u_c = \frac{1}{\mu}\left(\frac{G}{4}\right)\left(R^2 - \frac{8}{3}R_c^{1/2}R^{3/2} - \frac{R_c^2}{3} + 2R_c R\right) \tag{31}$$

The flow through the core region thus can be expressed as

$$Q_{core} = \frac{\pi}{\mu}\left(\frac{G}{4}\right)\left(R^2 R_c^2 - \frac{8}{3}R_c^{5/2}R^{3/2} - \frac{R_c^4}{3} + 2R_c^3 R\right) \tag{32}$$

The expression of flow through the peripheral region is given in equation (33)

$$Q_{peripheral} = \frac{\pi}{\mu}\left(\frac{G}{4}\right)\left[\frac{R^4}{2} - R_c^2 R^2 + \frac{13}{42}R_c^4 + \frac{8}{3}R_c^{5/2}R^{3/2} - \frac{8}{7}R_c^{1/2}R^{7/2} - 2R_c^3 R + \frac{2R_c^3 R}{3}\right] \tag{33}$$

Thus the expression of flow through the blood vessel is expressed as

$$Q = \frac{1}{42}\left[\begin{array}{l}\left\{21R_1^4 - R_c^4 + 28R_cR_1^3 - 48\sqrt{R_c}R_1^{7/2}\right\}l + 84\left\{R_1^4 + R_cR_1^3 - 2\sqrt{R_c}R_1^{7/2}\right\}\left\{KI_1^2\right\}\left\{l - l_1\right\} \\ -84\left\{R_1^4 + R_cR_1^3 - 2\sqrt{R_c}R_1^{7/2}\right\}\left\{\delta S_1' L_1 + \delta S_2' L_2\right\}\end{array}\right] \tag{34}$$

The expression of wall shear stress through the blood vessel is given in equation (35)

$$\tau = \left[\begin{array}{l}\left\{R_1 + \frac{21R_1^5}{R_c^4} + \frac{28R_1^4}{R_c^3} - \frac{48R_1^{9/2}}{R_c^{7/2}}\right\}l + \left\{R_1 + \frac{105R_1^5}{R_c^4} + \frac{112R_1^4}{R_c^3} - \frac{216R_1^{9/2}}{R_c^{7/2}}\right\}\left\{KI_1^2\right\}\left\{l - l_1\right\} - \\ \left\{\delta S_1' L_1\left\{1 + \frac{\pi^2}{L_1^2}(d_1 + L_1/2)\right\} + \delta S_2' L_2\left\{1 + \frac{\pi^2}{L_2^2}(d_2 + L_2/2)\right\}\right\}\left\{\frac{216R_1^{9/2}}{R_c^{7/2}} - \frac{105R_1^5}{R_c^4} - \frac{112R_1^4}{R_c^3} - 1\right\}\end{array}\right] \tag{35}$$

Resistance to flow at the wall for the flow of blood is given by

$$\lambda = \left[\frac{4\mu}{\pi}\right]\int_0^l \left[\frac{R^4}{2} - \frac{R_c^4}{42} - \frac{8R^{7/2}R_c^{1/2}}{7} + \frac{2RR_c^3}{3}\right]^{-1} dz$$

The resistance to flow at the wall for the flow of blood in uniform portion of blood vessel is thus

$$\lambda_0 = \left[\frac{4\mu}{\pi}\right]\int_0^l \left[\frac{R_1^4}{2} - \frac{R_c^4}{42} - \frac{8R_1^{7/2}R_c^{1/2}}{7} + \frac{2R_1R_c^3}{3}\right]^{-1} dz$$

Thus the resistance parameter for the flow of blood in the blood vessel is expressed as

$$\lambda' = 2L_1'\left[1 + 8\delta S_1' + \frac{16\sqrt{R_c}}{7} - \frac{R_c^4}{21} - \frac{4R_c}{3} - 8\delta S_1'\sqrt{R_c}\right] + \left(l - L_1'\right) +$$

$$2\left[\left(1 - \frac{4R_c}{3} + \frac{R_c^4}{81} + \frac{16\sqrt{R_c}}{7}\right)\left(l - l_1' - L_2'\right) - \left(\frac{4K + 4R_cK - 8K\sqrt{R_c}}{3}\right)\left\{\left(d_2' + L_2' - l_1'\right)^3 - \left(d_2' - l_1'\right)^3\right\}\right]$$

$$-\frac{2}{3}\left(\frac{4K + 4R_cK - 8K\sqrt{R_c}}{3}\right)\left\{\left(l - l_1'\right)^3 - \left(d_2' + L_2' - l_1'\right)^3 + \left(d_2' - l_1'\right)^3\right\} +$$

$$2L_2'\left[1 + \frac{16\sqrt{R_c}}{7} + \frac{R_c^4}{21} - \frac{4R_c}{3} - 8\delta S_2'\left(\sqrt{R_c} - 1\right)\right] \quad (36)$$

Effect of various parameters on the flow of blood in stented blood vessels:

In order to get a physiological insight into the effect of stenosis on the wall shear stress, flow rate and resistance parameter against $\delta S`_1$ or $\delta S`_2$or both, for different values of wall exponent parameter K, i.e., K>0(divergence of artery), K=0(uniform portion of capillary) and K<0(convergence of veins), computations are made for Power law model, Bingham Plastic model and Casson model and are shown in the sections below.

Analysis:

In all the three models developed, we observe that as the height of stenosis increases in the blood vessels, wall shear stress also steadily increases for different values of wall exponent parameter, i.e., K>0(divergence of artery), K=0(uniform portion of capillary) and K<0(convergence of veins).The mean arterial blood pressure (MAP) in arteries is around 100 mm. Hg, in the capillaries the MAP is 25 mm Hg and in the veins and venae-cavae its mean pressure falls progressively to about 0 mm Hg in the systemic circulation. Similarly in the pulmonary circulation the MAP is 16 mm Hg, whereas, in the pulmonary capillary it is 7 mm Hg and in the pulmonary veins its mean pressure falls progressively to about 0 mm Hg like in systemic circulation. All the above three models depict the physiological conditions like K>0, K=0, and K<0.

Analysis:

In all the three models developed, we observe that as the height of stenosis increases in the blood vessels, flow rate steadily decreases for different values of wall exponent parameter, i.e., K<0(convergence of artery), K=0(uniform portion of artery) and K>0(divergence of artery).

In all the three models developed, we observe that as the height of stenosis increases in the blood vessels, resistance parameter steadily increases for different values of wall exponent parameter, i.e., K<0(convergence of artery), K=0(uniform portion of artery) and K>0(divergence of artery).

Casson Model:

Table 11: Variation of λ` against δS`$_2$ for K= -0.001, 0, 0.001

λ`	δS`$_2$ K= -0.001	K=0	K=0.001
.027	11.44136168	11.69193888	11.54546748
.034	11.44232376	11.69290096	11.54642956
.040	11.4431484	11.6937256	11.5472542
.046	11.44397304	11.69455024	11.54807884
.053	11.44493512	11.69551232	11.54904092
.060	11.4458972	11.6964744	11.550003
.067	11.44685928	11.69743648	11.55096508

Table 12: Comparative analysis of various parameters on Non-Newtonian models for the flow of blood in blood vessels

	K<0 (Artery)			K=0 (Capillary)			K>0 (Veins)		
	Flow rate	Shear stress	Resistance parameter	Flow rate	Shear stress	Resistance parameter	Flow rate	Shear stress	Resistance parameter
Power fluid	39.14167	12.59833	5.15214	39.74167	27.74833	5.004	40.35667	42.899	4.85581
Bingham plastic fluid	73.215	43.66967	41.78829	75.135	41.77	41.01192	74.175	43.72333	42.56403
Casson fluid	33.15833	42.36583	11.44407	38.552	50.1345	11.69465	43.77833	57.90282	11.54818

Discussion:

Wall shear stress is an important factor in the study of blood flow. Accurate predictions of the distribution of the wall shear stress are particularly useful for the understanding of the effect of blood flow on endothelial cells. However, the flow rate in the arteries is affected much compared to veins, as arteries are resistance vessels, whereas veins are capacitance vessels. In hypertensive patients, the sustained increased pressure in arteries will lead to remodeling of the blood vessels and heart, especially in the resistance vessels where the pressure is very high. Arteries tend to become less elastic and stiff. In the models discussed, the trends observed show that as the stenosis increases there is an increase in the MAP in the resistance vessels which may lead to remodeling of the arteries. The remodeling is not prominent in capillaries and veins, where the resistance to flow is least compared to arteries.

In the models developed above, we observe that Casson model and Power law fluid model well suits for the physiological data (Table 12).

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