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Extended Haessler Heuristic Algorithm for Cutting Stock Problem: a Case Study in Film Industry

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Abstract: The cutting stock problem is an important problem in the field of combinational optimization. In classic versions of this problem, the aim is to find a solution for cutting some equal-width mother rolls into smaller parts with given width so that the amount of trim-loss become as small as possible. There are plenty of works which addressed the classic cutting stock problem. However in real world applications there are usually some constraints which make the problem different form classical version and make it harder to solve. In this paper we investigate a special cutting stock problem in film industry and show that it can not be solved by using previous methods. We propose a new hybrid method which combines previous methods and takes into account the constraints of this special problem. We applied our method on real data from a film production factory. The experiments show that our method can solve this problem efficiently and outperforms the previous methods.

Key words: Cutting Stock Problem, Linear Programming, Heuristic methods

INTRODUCTION

The cutting stock problem is an important problem in the field of combinational optimization which has many applications in industry. In one dimensional case, we have an abundant number of stock rolls and some orders. The problem is to cut the stock rolls into orders with given demand for each order. In film industry, the small components are called order list and stock rolls are called mother rolls. The primary objective of cutting process is to minimize trim-loss, so this problem is called trim-loss problem too. This problem contains bin packing problem, which is known to be strongly NP-hard (Garey, M.R. and D.S. Johnson, 1979), so it is clearly a hard problem. Each mother roll is cut to some smaller components by using a combination of small widths. Each such combination is called a cutting pattern. For example we can cut a mother roll of width 6600mm to components with widths 1200, 1200, 650, 780, 780, 1100mm and we have 290 mm trim loss. In figure 1 an instance of this problem is depicted.

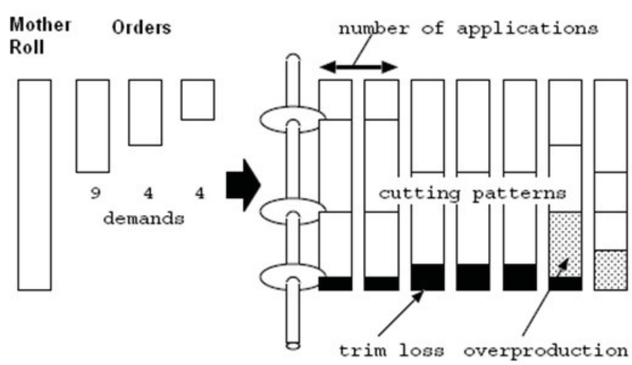

Fig 1: An instance of one-dimensional cutting stock problem

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In figure 1 there are four cutting patterns which are used 2, 3, 1, 1 times respectively. After applying each cutting pattern, the machine should be stopped for a short time to be adjusted for next cutting pattern; this time is called setup time. The smaller number of patterns used, the less setup time (and less machine stops) is needed. So in relevant industries, in addition to minimizing the trim-loss, the number of pattern changes is important too. As can be seen in figure 1, in addition to order list, some other widths are produced too. Although these widths are not in order list, but producing them is better than having a lot of wastes. These widths may be needed in future orders. We call these widths auxiliary widths and we should try to reduce their production in cutting process.

A formal representation of the cutting stock problem is presented in (Gilmore, P.C. and R.E. Gomory, 19961). Each cutting pattern is a feasible combination of ordered widths that should be cut from mother roll and is shown by $P_j = \{a_{1j}, a_{2j}, \dots, a_{mj}\}$ in which m is number of ordered widths, and $a_{ij} \in Z_+$ is number of cuts with i-th width in j-th pattern. Regarding to the fact that sum of widths in each pattern should not exceed the width of mother roll, so we should have:

$$\sum_{i \in \{1,2,\dots,m\}} a_{ij} w_i \leq W$$

In which W is the width of mother roll and w_i is the width of i-th order. If P_j pattern satisfy the above condition, we call it a feasible pattern. Let S be the set of all feasible patterns, the cutting stock problem is to determine $\Pi = \{P_1, P_2, \dots, P_{|\Pi|}\}$ and usage frequency of each pattern $X = \{x_1, x_2, \dots, x_{|\Pi|}\}$ so that cut numbers from mother rolls is minimized, and all order demands be fulfilled. It is obvious that minimizing the use of mother rolls is equivalent to minimizing trim-loss. Therefore we can formulate the problem as follows:

$$minimize\ f(\Pi, X) = \sum_{P_j \in \Pi} x_j$$

$$Subject\ to\ \sum_{P_j \in \Pi} a_{ij} x_j \geq d_i \qquad for\ i \in \Pi$$

$$x_j \in Z_+ \qquad for\ P_j \in \Pi$$

In which d_i is the demand of i-th order.

Related Works:

A classic approach for solving the cutting stock problem is using integer linear programming methods. Having all feasible patterns, the problem is to find a combination of uses of these patterns in order to minimize an objective function. Usually there are too much feasible patterns. Let m be the number of ordered widths and k be the average number of components in patterns, the number of feasible patterns is $\frac{m!}{k!(m-k)!}$. Solving linear programming problem with such excessive number of patterns is impractical.

Gilmore and Gomory (1963) suggested that instead of considering all feasible patterns, we can restrict ourselves to a few patterns and generate new ones as needed. They used column generation (Lubbecke, M.E. and J. Desrosiers 2005) method for generating new useful patterns. In each step a new pattern which improves the current solution is added to patterns list. Generating new column involves solving a knapsack problem (Silvano Martello and Paolo Toth, 1990). This procedure continues until no new pattern can be found that improves current solution. As a result, we have a few patterns and therefore solving the integer linear programming problem would be easier. At the end the integer linear programming problem is solved and the usage frequency of each pattern is obtained. The number of patterns in solution is about the number of ordered widths.

Another approach is using sequential heuristic methods (Haessler, R.W., 1971; Haessler, R.W. and P.E. Sweeny, 1991; Haessler; R.W., 1988). In these methods, in each step a new pattern is generated. In each step we have a partial problem which involves remaining demands. A new pattern is selected by using some special conditions and will be added to solution at highest inspiration level. Accordingly the number of order demands is reduced and the procedure continues until there are no more demands. Though integer linear programming approach guarantees minimizing trim-losses, but it can not be used in many of the real world problems.

In many industries (and especially in film industry) the number of machine stops for changing patterns is very important. Integer linear programming usually needs a plethora of cessations which is not desirable. Furthermore cutting machine has a fixed number of knives that restricts the number of components in each pattern. Adding this constraint to the problem makes the column generation problem very difficult to solve.

In the next section we explain the problem in film industry, and then we propose a new solution for solving it which is a combination of sequential heuristic methods and integer linear programming.

Problem Statement:

The cutting stock problem in film industry has some characteristics that make it different from classical cutting stock problem and much harder to solve. These characteristics are:

- Restricted number of cutting knives: Cutting machines always have a fixed number of knives, which we can use a subset of them. So cutting patterns that need more knives are not applicable. For example if our cutting machine has 10 knives, we can not use patterns with 11 or more components. If we use the integer linear programming, the corresponding column generation will be very difficult to solve. In fact the sub problem is not a knapsack problem anymore. So we can not use the integer linear programming.
- The number of machine setups: in addition to trim-loss, the number of patterns in solution should be reduced to avoid many machine stops. Bellow and Scheithauer (Belov, G.and G. Scheithauer, 2007) proposed a non-linear programming method that considers setup times in objective function.
- Delivery tolerance: in many cases we can change the number of demands of each width to some degree. For example if have an order for 100 rolls of a special width, we can deliver 90-110 rolls to customer instead, if our delivery tolerance is 10 percent. Considering this tolerance may help us to get better solutions. For example we can remove a pattern with too much trim-loss as long as produced components satisfy the specified tolerance; or we can add a component to a pattern which reduces trim-loss while we do not exceed the specified tolerance.
- Auxiliary widths: in some cases making a proper solution with an acceptable amount of trim-losses may be impossible. For example if width of mother roll is 1000mm and order list includes widths 300 and 200, each pattern has at least 100mm trim-loss. In this case adding an extra width of 75mm will be useful. However we should avoid excessive use of these auxiliary widths. These widths are widths which probably are needed in future. We provide a priority for each auxiliary width which determines our preference about using each of them.

Regarding to these constraints we devised a new algorithm which comes up with them and generates a good solution. In the next section we present our algorithm.

Proposed Algorithm:

Our proposed algorithm is a combination of sequential heuristic method and integer linear programming. First by using sequential heuristic method we find a solution, then we feed the obtained patterns to the integer linear programming and if we get better results from that, we use it; otherwise we use the former solution.

We used the sequential heuristic approach to define more complex objective functions and incorporate more constraints into problem. Sequential heuristic method is a local search method. In this method we produce a cutting pattern in each step. The new pattern should have small trim-loss and reduce the objective function. Furthermore we should preserve some small widths for consecutive steps, to avoid generating widths that can not be used in later steps. After producing each pattern, the new pattern will be added to solution at highest inspiration level and the remaining demand will be updated correspondingly. The procedure continues to satisfy all demands in specified tolerance.

Our proposed algorithm which is an extension of Haessler (Silvano Martello and Paolo Toth, 1990; Haessler, R.W., 1971; Haessler, R.W. and P.E. Sweeny, 1991) method and takes into account the special characteristics of our problem, is as follows:

1. Compute *descriptors* of the order requirements to be scheduled.
   The first descriptor is an estimate of the number of mother rolls needed to satisfy remaining order requirements $\sum_i \frac{d'_i w_i}{W}$, where d'_i is the number of rolls needed to be scheduled for the *i*th order width.
   The second descriptor is the average number of components in each pattern and is $\sum_i d'_i / [\sum_i d'_i w_i / w]$.
2. Set *aspiration levels* for the next cutting pattern to enter the solution by using the descriptors.
   The aspiration level is a goal that must be met for each of the important characteristic for next cutting pattern. The characteristics are:

- The maximum allowable trim loss Δ_{max}
- The maximum and minimum number of components in the cutting pattern, N_{min}, N_{max}
- The minimum number of pattern using $N_{pat,min}$
- Haessler suggested that Δ_{max} ranges from *0.006W to 0.03W*. The value of $N_{pat,min}$ ranges from 0.5 to 0.9 of the first descriptor. The value of N_{max} is the number of machine knives. N_{min} is usually one less than the second descriptor, computed in step 1.

3. Find the set of all cutting patterns that meets aspiration levels(S).
   A found pattern is a column vector P_j with nonnegative integer elements P_{ij}, where

- $W - \sum_i P_{ij} w_i \leq \Delta_{max}$
- $N_{min} \leq \sum_i P_{ij} \leq N_{max}$
- $\frac{d'_i}{P_{ij} N_{pat,min}} \geq 1 \quad for\ all\ P_{ij} \geq 0$

   First constraint keeps the trim loss within the allowable bounds. Second constraint guaranties that the number of components would not exceed number of machine knives and be greater than N_{min}. Third constraint limits the times a particular width can appear in cutting patterns in order to keep number of any order produced within allowable bounds.

4. If $S = \emptyset$, reduce inspiration levels in following order:

- If $N_{pat,min}$ is greater than one, reduce it.
- Use auxiliary widths according to their priority.

- If all auxiliary widths are used, increase Δ_{max}

5. For $S \neq \emptyset$, do:

- Select $P_j \in S$ with the highest possible level (number of times the pattern used).

  Highest possible level obtained from remaining requirements. Let d_i' be remaining requirement of *i*th order width and P_{ij} be number of times it is used in pattern. Highest possible level is $\min\{\frac{d_i'}{P_{ij}}\}$.

- Add P_j and the number of times it is used to solution and reduce the order requirements.

  Assume that x_j is the number of times pattern P_j is used. Remaining order requirement of *i*th order is:

  $$d_i'(new) = d_i' - x_j * P_{ij}$$

- If all requirements are satisfied, compare current solution with ones in solutions list (AL). If it is not worst among all of them, then add it to AL.
  One solution is worse than other solution if it is worse in trim loss, number of patterns and use of auxiliary width. For example a solution that has 1000mm trim loss, 7 patterns and 10 auxiliary width used, is worse than a solution with 1100mm trim loss, 9 patterns and 2 auxiliary widths.

- Having current A and order remaining requirements go to 1.

- Remove P_j from A and S. Then set the value of d_i' to values before using pattern P_j

6. Solve corresponding integer linear programming problem for each of the solutions in solution list (AL). For each solution, if the solution of integer liner programming is better, replace it with LP solution in AL. For each solution, the cutting patterns used in it will be input parameters for linear programming problem. It is obvious that the number of patterns in new solution does not exceed the patterns in former one. So if new solution be better by trim losses, then we can use it instead of former one.

7. For each solution in AL, calculate cost function and return the minimum cost solution as output.
   The cost function includes trim loss, number of pattern and number of auxiliary widths used. We can define proper coefficients for each of these values and calculate the cost function.

Experiments:

In this section we present the results of applying our method on some real samples in a film production factory. In this factory, each pattern change costs about 200mm of trim-loss. Cost of using each auxiliary width is about 40mm of trim-loss. The cost function is sum of trim-losses, number of pattern changes timed by 200, and the number of used auxiliary widths timed by 40. In this factory, the width of mother roll is 6480mm. The cutting machine has 10 knives and delivery tolerance is 10 percent. We show the results obtained from each of following methods:

1. Manual method based on human experience
2. Integer linear programming ILP method of Gilmore & Gomory (ILP)
3. Sequential heuristic method of Haessler (SHH)
4. Our proposed algorithm

Experiment 1. Ordered widths are as shown in table1:

Table 1: Orders list of experiment 1

Order Width	Demand	Is auxiliary width
400	12	No
510	100	No
560	8	No
600	72	No
680	24	No
720	62	No
760	40	No
850	8	No
900	8	No
960	14	No
1020	22	No
1100	8	No
1200	14	No

From table1 it is obvious that we can not have any auxiliary width. Now we present the solutions generated by each of the methods.

1. Manual method

Table 2: Result of manual method for experiment 1

Pattern No	Used Frequency	Pattern widths
1	14	510 510 720 760 760 960 1020 1200
2	12	400 600 600 600 600 720 720 720 720 760
3	8	510 510 510 510 600 680 680 680 850 900
4	8	510 510 510 510 510 560 600 600 1020 1100

The trim loss is 1840mm and the number of patterns is 4, therefore the cost value is:

1840 + 4*200 = 2640

2. ILP

Table 3: Result of ILP method for experiment 1

Pattern No	Used Frequency	Pattern widths
1	1	400 400 400 400 400 400 400 400 400 400 400 400 400 600 680
2	1	510 510 510 510 560 560 560 560 1100 1100
3	6	1020 1020 1020 1020 1200 1200
4	12	510 510 510 510 510 510 510 510 680 760 960
5	7	720 720 720 720 720 720 720 720 720
6	10	600 600 600 600 600 600 600 760 760 760
7	2	400 680 900 900 900 900 900 900
8	2	560 560 600 680 680 850 850 850 850
9	1	720 960 960 960 960 960 960
10	2	680 680 720 1100 1100 1100 1100
11	1	400 600 680 1200 1200 1200 1200

The solution has no trim loss and number of used pattern is 11. Note that the numbers of components in first and 4th pattern are greater than 10, the number of machine knives. So this solution is impossible and the cost function is ∞.

3. SHH

Table 4: Result of SHH method for experiment 1

Pattern No	Used Frequency	Pattern widths
1	22	510 510 510 600 600 600 680 720 720 1020
2	9	510 510 720 720 760 760 760 760 960
3	6	400 400 510 510 560 600 1100 1200 1200
4	2	510 510 560 680 760 760 900 900 900
5	2	850 850 850 900 960 960 1100
6	1	850 850 960 1200 1200

Trim loss is 1840 and the number of patterns is 6. So the cost value is

1840 + 6 * 200 = 3040

4. Proposed algorithm

Table 5: Result of proposed method for experiment 1

Pattern No	Used Frequency	Pattern widths
1	1	510 560 560 600 850 850 850 850 850 850
2	1	410 410 410 510 510 510 680 850 1100 1100
3	2	400 510 680 850 900 960 960 1200
4	6	400 510 560 600 900 1100 1200 1200
5	10	510 510 720 720 760 760 760 760 960
6	21	510 510 510 600 600 600 680 720 720 1020

Trim loss is 530 and the number of patterns is 6. So the cost function is
530 + 6 * 200 = 1630

The results are shown in following table:

Table 6: Results of methods for experiment 1

Method	Trim loss	Number of Patterns	Number of auxiliary widths	Cost value
Manual	1840	4	0	2640
ILP	0	11 (two of them infeasible)	0	∞
SHH	1840	6	0	3040
Proposed	530	6	0	1630

Table 6 shows that the proposed algorithm gets the best result. Although the SHH gets worse result than manual method result, but it has broad trim loss (1840mm in last pattern) that can be used in future. The trim losses of manual methods are small in width and can not be used.

Experiment 2. Ordered widths are as shown in table 7:

Table 7: Orders list for experiment 2

Order Width	Demand	Is auxiliary width
510	54	No
600	18	No
680	61	No
720	17	No
730	33	No
760	14	No
900	12	No
950	12	No
1020	32	No
1100	30	No
1140	22	No
1200	32	No
1356	54	No

In this experiment no auxiliary width can be used too. Now we present the solutions generated by each of the methods.
1. Manual method

Table 8: Result of manual method for experiment 1

Pattern No	Used Frequency	Pattern widths
1	18	510 680 1200 1356 1356 1356
2	12	510 510 510 720 900 950 1140 1200
3	6	1020 1020 1100 1100 1100 1100
4	5	600 680 720 730 730 730 1140 1140
5	3	600 600 600 680 680 1090 1090 1090
6	2	600 600 760 1100 1100 1100 1200
7	4	680 680 1020 1020 1020 1020 1020
8	6	680 680 680 680 730 730 730 760 760

The trim loss is 1736mm and the number of patterns is 8, then the cost function value is:
1736 + 6*200 = 3336
2. ILP

Table 9: Result of ILP method for experiment 2

Pattern No	Used Frequency	Pattern widths
1	3	600 680 680 680 680 680 730 730 1020
2	4	510 510 510 510 510 510 510 510 600 900 900
3	2	600 600 600 600 600 600 600 1140 1140
4	1	720 720 720 720 720 720 720 720 720
5	1	510 510 510 510 510 510 510 510 1200 1200
6	6	680 680 720 1100 1100 1100 1100
7	12	510 760 1140 1356 1356 1356
8	6	510 950 950 1356 1356 1356
9	8	1020 1020 1020 1020 1200 1200
10	5	730 730 730 730 730 730 900 1200
11	2	600 720 760 1100 1100 1100 1100
12	6	680 680 680 680 680 680 1200 1200
13	2	510 680 730 1140 1140 1140 1140

The solution has 36mm trim loss and number of used pattern is 13. Note that the number of components in second pattern is greater than 10, the number of machine knifes. So this solution is impractical and the cost function is ∞.

3. SHH

Table 10: Result of SHH method for experiment 2

Pattern No	Used Frequency	Pattern widths
1	18	510 510 510 600 680 680 680 1100 1200
2	14	730 1020 1020 1140 1200 1356
3	9	730 730 950 1356 1356 1356
4	6	720 760 760 900 1100 1100 1140
5	3	680 680 720 720 950 1356 1356
6	2	720 760 900 900 1020 1020 1140
7	1	680 720 720 730 900 1356 1356
8	1	900 1356 1356 1356 1356

Trim loss is 662 and the number of patterns is 8. So the cost function is
$662 + 8 * 200 = 2262$

4. Proposed algorithm

Table 11: Result of proposed method for experiment 2

Pattern No	Used Frequency	Pattern widths
1	1	720 760 760 760 760 1356 1356
2	6	680 720 900 900 950 950 1356
3	9	510 600 600 680 680 720 730 760 1200
4	15	510 510 510 680 1020 1020 1100 1100
5	22	680 730 1140 1200 1356 1356

Trim loss is 998 and the number of patterns is 5. So the cost function is
$998 + 5 * 200 = 1998$

The results are shown in following table:

Table 12: Result of methods for experiment 2

Method	Trim loss	Number of Patterns	Number of auxiliary widths	Cost value
Manual	1736	8	0	3336
ILP	36	13 (one of them infeasible)	0	∞
SHH	662	8	0	2262
Proposed	998	5	0	1998

Table 12 shows that the proposed algorithm gets the best result.

Experiment 3. Ordered widths are as shown in table 13:

Table 13: Orders list for experiment 3

Is auxiliary width	Demand	Order Width
NO	15	680
NO	30	1120
NO	20	1140
NO	25	1150
NO	54	1200
NO	48	1250
NO	6	1550
YES	Priority1	720
YES	Priority2	600

As shown in Table 13 we can use two auxiliary widths if needed. We prefer to use width of 720mm, because one of our customers has ordered it and we should deliver it about two months later (not now). Now we present the solutions generated by each of the methods.

1. Manual method

Table 14: Result of manual method for experiment 3

Pattern No	Used Frequency	Pattern widths
1	6	1250 1250 1200 1200 1550
2	8	1250 1250 1250 1250 720 720
3	10	1200 1200 1140 1140 1120 680
4	10	1200 1200 1150 1120 1120 680
5	6	1200 1200 1150 1150 1150 600
6	2	1250 1120 1120 1120 1120 720

The trim loss is 840 mm, the number of patterns is 6 and 24 auxiliary widths used, so the cost function value is:

$$840 + 200 * 6 + 24 * 40 = 3000$$

2. ILP

Table 15: Result of ILP method for experiment 3

Pattern No	Used Frequency	Pattern widths
1	4	680 1120 1120 1120 1200 1200
2	3	1120 1120 1120 1120 1120
3	5	680 1140 1140 1140 1140 1200
4	7	680 1150 1150 1150 1150 1200
5	7	1200 1200 1200 1200 1200
6	10	1250 1250 1250 1250 1250
7	3	1120 1120 1140 1550 1550

The solution has 8660mm trim loss and number of used pattern is 7. so the cost function value is:

$$8660 + 7*200 = 10060$$

3. SHH

Table 16: Result of SHH method for experimrnt 3

Pattern No	Used Frequency	Pattern widths
1	8	1200 1200 1200 1250 1250
2	10	680 1120 1140 1140 1150 1250
3	6	680 1120 1120 1150 1150 1150
4	15	1120 1120 1120 1550 1550

Trim loss is 8210mm and the number of patterns is 4. So the cost function is

$$8210 + 4 * 200 = 9010$$

4. Proposed algorithm

Table 17: Result of proposed method for experiment 3

Pattern No	Used Frequency	Pattern widths
1	8	1250 1250 1250 600 710 710 710
2	10	1140 1140 1150 1200 1250 600
3	6	1200 1200 1250 1250 1550
4	15	680 1120 1120 1150 1200 1200

Trim loss is 330 mm, the number of patterns is 4 and 42 auxiliary widths used. So the cost function value is
330 + 4 * 200 + 42*40 = 2810

The results are shown in following table:

Table 18: Result of methods for experiment 3

Method	Trim loss	Number of Patterns	Number of auxiliary widths	Cost value
Manual	840	6	24	3000
ILP	8660	7	0	10060
SHH	8210	4	0	9010
Proposed	330	4	42	2810

Table 18 shows that the proposed algorithm yields the best result.

Conclusion and Future Work:

In this paper we described cutting stock problem; a special type of optimization problems which frequently occur in real world industrial applications. Our aim was to get a practical and efficient solution for an instance of the general problem in film industries. We specified some characteristics that make previous methods impractical for solving this special instance of problem. So we presented a new hybrid method which combines previous methods and uses best features of each of them to come up with this special kind of problem. Experiments show that proposed method gives better results comparing with previous methods.

In film industries, sometimes 2 stage solutions are used, in which mother rolls are cut into broader widths and then these broad widths are fed to another machine which cuts them into widths needed by customer. One possible improvement to our method is to extend it to support the 2 stage solutions, which probably may improve the efficiency of method and result in better solution.

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REFERENCES

Belov, G.and G. Scheithauer, 2007. "Setup and open-stacks minimization in one-dimensional stock cutting", INFORMS Journal on Computing, 19(1): 27-35.

Garey, M.R. and D.S. Johnson, 1979. "Computers and intractability- a guide to the theory of NP-completeness", W.H.Freeman and Company.

Gilmore, P.C. and R.E. Gomory, 19961. “A linear programming approach to the cutting-stock problem", Operations Research, 9: 849-859.

Gilmore, P.C. and R.E. Gomory, 1963. “A linear programming approach to the cutting-stock problem", Operations Research, 11: 863-888.

Haessler, R.W., 1971. "A heuristic programming solution to a nonlinear cutting stock problem", Management Science, 17: 793-802.

Haessler, R.W. and P.E. Sweeny, 1991. “Cutting stock problems and solution procedures ”, European journal of Operational Research, 54: 141-150.

Haessler; R.W., 1988. “Selection and Design of Heuristic Procedures for Solving Roll Trim Problems”, Management science, 34(12): 1460- 1471.

Lubbecke, M.E. and J. Desrosiers 2005. “Selected topics in column generations”, Operation Research, 6: 1007-1023.

Silvano Martello and Paolo Toth, 1990. "KNAPSACK PROBLEMS: Algorithms and Computer Implementations", JOHN WILEY & SONS.