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A New Practical Model to Trade-off Time, Cost, and Quality of a Project

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Abstract: The three interrelated and conflicting objectives of any project are time, cost, and quality. In today's competitive business environment, delivering projects in the least possible time, with maximum quality and minimum cost has got a critical issue for project managers. These objectives are dependent on the related features of the activities (i.e. time, cost, and quality) of that project. Since these features are also interrelated, different models have been introduced to define the relation among them. The existing relations in the literature propose that the quality of an activity varies dependently by its time decreasingly and therefore lack the generality that an activity can be executed with the least possible time and the best quality, although by spending more money. In this paper, first the general characteristics of a proper relation function are presented as six axioms. Then a new model to define the relation among features of an activity, practicable in real world projects is developed, that meets the stated characteristics. Then we developed a new mathematical trade-off model for the trio of objectives of a project based on the proposed model. To illustrate the applicability of the proposed model, a sample project is investigated, and the according time, Cost, and quality Pareto optimal front of the project is found using a recent version of the ε-constraint method. Since the Pareto front is three dimensional, to provide insightful information that can help the managers in making trade-off decisions, quality, cost, and time contours of the Pareto surface are also provided.

Key words: ε-Constraint Method, Multi-Objective Decision Making, Project Scheduling, Time, Cost, Quality Trade-Off

INTRODUCTION

The American National Standard Institution (ANSI) defines project management as "*the application of knowledge, skills, tools, and techniques to project activities to meet project requirements*", PMI (2004). Project scheduling, an integral part of project management, is intended to balance the competing objectives of a project, while maintaining the project scope. The three fundamental objectives of project management are completion time (hereafter called time), total cost (hereafter called cost), and quality (Demeulemeester and Herroelen (2000)). The trio of project scheduling (see Figure 1) and consequently the trade-off among them has been the subject of several studies, so far.

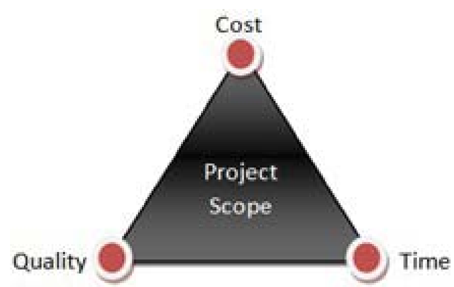

Fig. 1: The Trio of Project Objectives

The critical path method (CPM) is a fundamental quantitative technique developed for project management. Assuming deterministic activity times, CPM determines the minimum time needed to complete the project. In the management of a project, it is often possible to compress the duration of some of the activities at an additional expense in order to reduce the total project's duration, and generally there is a due date (or in some cases called soft deadline) for project completion; So a decision problem considered in the project management

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literature is to determine the activities for crashing and also the extent of crashing. By assuming that the direct cost (hereafter called cost) of an activity varies with time (limited by normal and crash times), mathematical programming models were developed to minimize the project direct cost. The problem is known as *continuous time/cost trade-off problem* in the literature. This problem was first studied by Kelly and Walker (1959). He assumed a linear relation between time and cost of an activity and offered a mathematical model, as well as a heuristic algorithm for solving the problem. Fulkerson (1961) presented a solution method to find the curve of time-cost trade-off, or in other words for each time realization of the project, the best time, and cost of each activity to minimize the total cost of the project subject to the given due-date were found.

Several other forms of activity cost functions have been studied, too. Moder *et al.* (1983) considered a convex function, and by a very simple trick approximated the convex cost-duration curve by linear segments, and then tried to solve the simplified problem. Kapur (1973) offered a labeling algorithm for the case where the cost-duration functions are quadratic and convex. Salem and Elmaghraby (1984) treated the general convex case and seek a satisfying answer, in which the project is compressed to a desired completion time with specified tolerable relative error. They constructed the optimal first degree interpolating linear spline that guaranteed such maximal error and considered various possible refinements. On the other hand, concave functions have also been studied. Moder *et al.* (1983) approximated the concave curve by a piece-wise linear one through employing a nonnegative integer variable. The idea of approximating the concave cost-duration curve by linear segments has been used by Falk and Horowitz (1972) in a branch-and-bound procedure.

In most practical cases, resources are available in discrete units, such as a number of machines, a number of workers and so on (Demeulemeester and Herroelen (2000)). This kind of problem is called multi-execution mode for activities or *discrete time-cost tradeoff problem* in the literature, and the best execution mode (time, cost) of the activities should be determined to optimize an objective subjected to some constraints. The literature of discrete time-cost tradeoff problem is quite comprehensive. For an extensive review on this problem, we refer the readers to De (1994), and Vanhoucke and Debels (2007).

Babu and Suresh (1996) were the first who suggested that the quality of a completed project may be affected by project crashing. They assumed that quality of a project is a function of the quality of its activities, and also assumed that cost and quality of each activity varies linearly with activity completion time. For simplicity they adopt the continuous scale from Zero to One to specify quality attained by each activity. The overall project quality is a function of quality levels attained by the individual activities. In that paper, they developed optimization models involving the project time-cost-quality tradeoff which would assist in expediting a project. They developed three mathematical models, and in each of them one of these three objectives were optimized by assigning desired levels on the other two. In Khang and Myint (1999), model proposed by Babu and Suresh (1996) was applied to an actual cement factory construction project. The purpose was to evaluate the applicability of the method by highlighting the managerial insights gained, as well as pointing out key problems and difficulties may be faced. The problems investigated by Babu and Suresh (1996), Khang and Myint (1999) can be categorized in the class of *continuous time/cost/quality trade-off problem*.

In El-Rayes *et al*. (2005), for the first time, the *discrete time–cost-quality trade-off problem* was investigated. They used a real world example and suggested new functions to enable the consideration of construction quality in the time, cost, and quality optimization problem in construction industry. To estimate the project quality, they introduced some quality indicators, and used the weighed sum of the quality levels sassed by indicators as the project quality. GA (Genetic Algorithm) was used as the solution method. The discrete time–cost-quality trade-off problem was also investigated in Tareghian and Taheri (2006). Each activity was assumed to be executed in different mode. The duration and quality of each activity were assumed to be discrete, non-increasing functions of a single non-renewable resource, i.e. time. Different forms of quality aggregations and effect of activity mode reductions were also investigated. Pollack-Johnson and Liberatore (2006) investigated the same problem, and proposed a model based on Analytic Hierarchy Method (AHP) to measure the overall quality for each alternative on hand. In Tareghian and Taheri (2007) a solution procedure based on scatter search combined with ideas borrowed from electromagnetism theory was presented to solve the discrete time, cost and quality tradeoff problem. Afshar *et al*. (2007) developed a new meta-heuristic based on multi-colony ant algorithm to solve this problem. They considered the quality of a project as the weighted sum of quality levels of its activities.

A multi-objective particle swarm (MOPS) for determining the best alternatives of a project's activities was studied in Rahimi and Iranmanesh (2008). A similar concept for Ideal Point in multi-objective optimization problems (Dynamic Ideal Point) was introduced and used in the initialization phase, as well as in the main part of the solution algorithm. In Iranmanesh *et al*. (2008) a meta-heuristic was developed based on a version of genetic algorithm specially adapted to solve multi-objective problems namely FastPGA to find the Pareto optimal front of the problem.

The research works in the field of time, cost, and quality trade-off, the subject of this research, can be categorized into two distinct categories.

1. Continuous trade-off problems: in this category, the relation among time, cost, and quality has been defined as continuous functions. In these works, one of the three variables (usually time) is considered to vary independently and the two other are defined as functions of that variable. For example, research works Babu and Suresh (1996), Khang and Myint (1999) are some examples.
2. Discrete trade-off problems: in this class, the relation among time, cost, and quality has been considered discrete. In other words, for each project activity, $i \in V$ different modes of execution are defined, and for each mode, distinct time, cost, and quality are associated. So to trade-off among the objectives, one execution mode is selected for each activity. Works El-Rayes *et al*. (2005), Tareghian and Taheri (2007), and Iranmanesh *et al*. (2008) are a few to cite

In cases which the project manager faces with a project in which there are alternatives for executing activities, and each alternative have distinct time, cost, and quality, discrete models are applicable. Project manager can select among these alternatives to optimize the trio of project objectives. The discrete models get impracticable in the projects that there are many activities, or there are many number of execution mode for each activity. In cases that the total number of the modes is very high (either because of high number of activities, or because of high number of mode per activity, or a combination), discrete models lose their applicability in two aspects:

- The definition of the problem parameters and data gathering for all modes of the project activities is not practical for project managers.
- The according problem gets very complex to solve.

On the other hand, continuous models are suitable in projects that project manager faces with many alternatives, and a continuous relation (or an approximation) among the time, cost, and quality of an activity can be defined. They are also applicable in cases that project activities are outsourced; therefore the time, cost, and quality of each activity can be bargained: "*how much it charges the company to reduce completion time of the activity or to improve its quality?*

Defining a realistic relation among completion time, cost, and quality of each activity, and specifying the parameters of the continuous model is not simple. An efficient relation among the three features of an activity has not been defined in the works available in literature. In Babu and Suresh (1996), and Khang and Myint (1999) only one independent variable (time) is defined and the two others (cost and quality) are determined consequently. In other words, by reducing the activity's time, its quality inevitably reduces; so it is impossible for a project manager to have a high quality activity with the lowest possible time. While in real world practice, it is possible to have an activity with the highest possible quality and the lowest time by spending more money. In this paper, we try to develop a practical model that defines a more realistic relation among time, cost, and quality of activities of a project, practicable for real world projects. This is actually the main motivation behind this research effort. The remaining parts of this paper are constructed as follows: in section 2, the proposed model for the relation among features of an activity is presented, based on the proposed model, a mathematical programming model of the trade-off of time, cost, and quality of a project is given in section 3; to illustrate the proposed model, a numerical example is investigated in section 4; and finally section 5 concludes our work.

The Proposed Model for the Relation among Features of an Activity:

The three features of an activity of a project i.e. quality, cost, and time are interrelated. As for other interrelated variables, the relation among these features can be defined in different ways by different functions. The variety may be due to the type of function, degree of freedom, and explicit or implicit nature of the function. As it was stated in the introduction, the defined relations existing in the literature lack the characteristics of real world situations. The problem is rooted in the degree of freedom considered in the relation, that is considered with one degree of freedom (only time can vary independently), which lead to a curve in space; while the real world, the true relation is a surface (with two degrees of freedom); two of the variables can vary independently, and the other is dependent on the value of these two. For instance, the direct cost needed to execute the activity is dependent on the time and quality level selected, or the quality of an activity can be determined by knowing both the time and budget level specified for it. The general characteristics of a *true surface* are stated as six axioms in the following section.

The General Characteristics of a True Cost/time/quality Surface:

In the proposed model, the relation among the three features of an activity is considered as a surface, taking time, and quality as independent variables, and cost as the dependant one, or by mathematical terms, cost of an activity *C*, is explicitly defined as $C = f(t, q)$ where *t*, and *q* are quality and time of the activity, respectively. For *C* to be a logically true function, the following characteristics should hold:

Axiom 1 *C* is defined on the set $[t_{crash}, t_{normal}] \times [q_{\min}\%, 100\%]$ where t_{crash} , t_{normal} , and $q_{\min}$ are the crash time (minimum technically possible time), normal time, and minimum quality defined for the activity, respectively.

Axiom 2 To decrease the time needed to complete an activity, more resources, or better resources should be used, that both lead to more expenditure. So, *C* is a non-increasing function of time; in other words, by reducing the time of an activity, its cost should not decrease; or by mathematical terms $\frac{\delta C}{\delta t} \le 0$

Axiom 3 To improve the quality of an activity, more resources, or better resources should be used, that both lead to more expenditures. So, *C* is a non-decreasing function of quality, or by mathematical terms $\frac{\delta C}{\delta q} \ge 0$; in other words, by increasing the quality of an activity, its cost should not decrease.

Axiom 4 Shortening an activity at long activity duration may be relatively cheap, while at short activity durations a much larger cost must be incurred for the same amount of reduction in the activity duration. So *C* is convex function of *t*. or by mathematical terms $\frac{\partial^2 C}{\partial t^2} \ge 0$. For instance it is costlier reducing the time of the activity from 200 days to 180 days, than reducing its time from 100 days to 80 days.

Axiom 5 Improving the quality of a high quality activity is relatively harder (costlier) than increasing its quality, when it is in poor quality conditions. So, *C* is convex function of *q*. or by mathematical terms $\frac{\partial^2 C}{\partial q^2} \ge 0$. For instance it is costlier improving the quality of an activity from 98% to 99% than improving it from 70% to 71%.

Axiom 6 Shortening a poor quality activity may be relatively cheap, while reducing the time of an activity of high quality (while maintaining its quality) is much costlier, and vice versa; by mathematical terms $\frac{\partial^2 C}{\partial q \partial t}, \frac{\partial^2 C}{\partial t \partial q} \le 0$. For instance it is costlier reducing the time of the activity with 90% quality from 200 days to 180 days, than reducing the same amount of time, when the quality is 50% (while maintaining its quality in both cases)

For the convexity assumptions (axioms 4, 5), it is worth noting that concavity has also been considered although rarely in the literature, for example see works Moder *et al*. (1983), and Kapur (1973). The concavity was explained with the economies of scale associated with long contractual periods; The concavity was explained with the economies of scale associated with long contractual periods; while the cost of an activity decreases by increasing its duration, the largest savings are realized at the longest durations. We challenge this assumption by two simple reasons. Consider a concave and a convex function between cost and time of an activity as illustrated in Figure 2, 3:

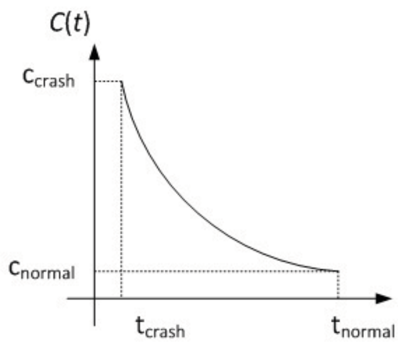

Fig. 2: Convex Function

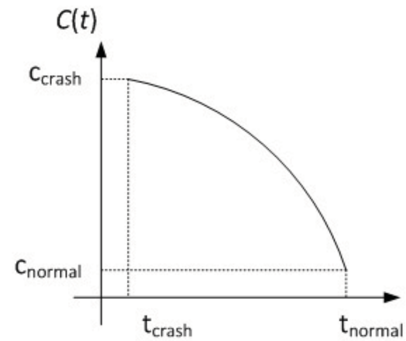

Fig. 3: Concave Function

1. The concave model is unable to justify the logic behind definition of *crash time*. While in the convex model, since the cost needed to reduce the time gets more and more when the time decreases, reducing the time less than a threshold time called *crash time* is extremely so high that is therefore neglected in practice.
2. In the concave model, as the time increases, the saving in cost increase such that, the cost can get even zero; so to avoid this, a threshold time is artificially defined called *normal time*. There is no logic behind the definition of this threshold. While in convex model, as the time increases, the saving of cost (because of the elongation of time) decreases such that it gets so low and negligible after some threshold called *normal time.*

Based on the above reasons, it can be inferred that the functions should be convex *at least* near the normal and crash times. Because a hybrid of concave and convex can also be considered, we restate the two axioms 4 and 5 as follows:

Axiom 4 C is convex function of t in the vicinity of $t = t_{crash}, t_{normal}$. or by mathematical terms $\left.\frac{\partial^2 C}{\partial t^2}\right| t = t_{crash}, t_{normal} \geq 0$

Axiom 5 C is convex function of q in the vicinity of $q = q_{\min}, 100\%$. or by mathematical terms $\left.\frac{\partial^2 C}{\partial q^2}\right| q = q_{\min}, 100\% \geq 0$

In real world projects, the number of activities in a project is very high, and since the function *C(t,q)* is a three dimensional surface, estimating this function for all activities is very difficult, even in some cases impossible. Therefore, in the following, we present features of a practical model that meets the general characteristics of a true surface *TC(t,q)* mentioned above, and is also simple for a project manager to apply.

The Practical Model Proposed for Surface C:

The followings are the assumptions of the proposed model:

1. For each activity, the normal activity duration denoted by *normal time,* corresponding to the most efficient work method used to perform activity, and the minimal duration of an activity denoted as its *crash time* are defined. Associated with normal time, *normal quality* is defined, which is always less than the ultimate possible quality (100%)

2. The cost of an activity can be categorized in three items:
   A. Cost of executing the activity with normal time and normal quality
   B. Cost of expediting the activity (reducing time)
   C. Cost of improving the quality of an activity (increasing quality)

3. By reducing the time of an activity (by spending money), its quality inevitably reduces as well. So, if we want the activity to maintain its quality (normal quality), the cost of the activity increases once more
4. If we want to have the activity with a quality better than normal quality, the cost of the activity increases. The amount of this increase is more when the time of the activity is less.

To implement the mentioned concepts, a mathematical model is defined using three functions:

1. The function $Q_T(t)$ determines the *regular quality of activity*, which is the corresponding quality for the selected time level $t = t_0$, if we do not want to spend money to maintain the normal quality of the activity, or improve its quality (refer to the assumption 4). This function $QT(t)$ is considered linear. To obtain this function, the quality of the activity in the normal time q_{norm}, and the quality of the activity in crash time q_{crash} should be determined (or equally by determining the marginal quality of reduction one unit of time ΔQ_T):

$$Q_T(t) = q_{norm} + \Delta Q_T.(t - t_{norm});\ \Delta Q_T = \frac{q_{norm} - q_{crash}}{t_{norm} - t_{crash}} \geq 0$$

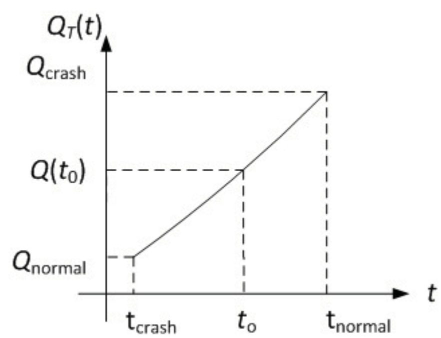

Fig. 4: Schematic Graph of Function $Q_T(t)$

2. The function $C_T(t)$ determines the cost of executing the activity with duration *t*. A linear relation is considered between *t* and $C_T(t)$. This line can be obtained by having normal cost C_{norm}, and the crash cost of activity C_{norm} (or equally by having the marginal cost of reducing the time of activity one unit of time $-\Delta C_T$):

$$C_T(t) = C_{norm} + \Delta C_T.(t - t_{norm});\ \Delta C_T = \frac{C_{norm} - C_{crash}}{t_{norm} - t_{crash}} \leq 0$$

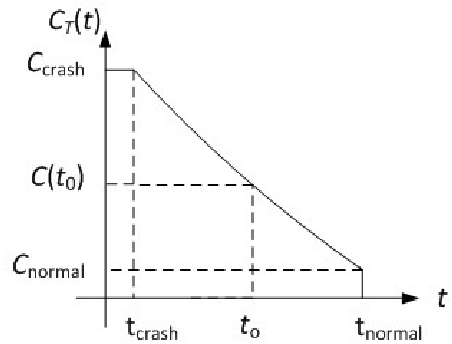

Fig. 5: Schematic Graph of Function $C_T(t)$

3. The function $C_Q(t,q)$ determines the cost of quality of the activity. Based on the value selected for q, this cost can be positive ($q > Q_T(t)$), negative ($q < Q_T(t)$) i.e. saving in money, and zero ($q = Q_T(t)$). To elaborate this function, let $t = t_0$, consider the function $C_Q(t = t_0, q)$, a linear function as follows:

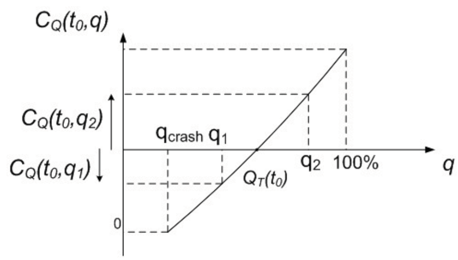

Fig. 6: Schematic Graph of Function $C_Q(t,q)$

The *regular quality of activity* is obtained using function $Q_T(t)$. If we want to have the activity with a higher quality $q = q_2 > Q_T(t_0)$, we should spend an amount of money equal to $C_Q(t_0, q_2)$, if we wish to have the activity with a lower quality than $Q_T(t)$, i.e. $q = q_1 < Q_T(t_0)$, an amount of money equal to $-C_Q(t_0, q_1) \geq 0$ is saved, otherwise $C_Q(t_0, q = Q(t_0)) = 0$. The slope of this line is increased linearly by decreasing in time. To construct this function, the cost of increasing one percent of quality in normal completion time of activity ΔC_Q^{norm}, and the cost of increasing one percent of quality in crash completion time of activity ΔC_Q^{crash} , should be determined; then we can interpolate the cost of increasing one percent of the quality for all $t \in [t_{crash}, t_{normal}]$. Since $(Q_T(t), 0)$ is a point of the line, the function $C_Q(q,t)$ can be defined as follows:

$$C_Q(t,q) = m_t.(q - q(t)); m_t = \left[\frac{\Delta C_Q^{norm} - \Delta C_Q^{crash}}{t_{norm} - t_{crash}}.(t - t_{norm}) + \Delta C_Q^{norm} \right]$$

The total cost of an activity is defined as the summation of the two mentioned costs:

$$TC(t,q) = C_T(t) + C_Q(t,q) =$$

$$C_{norm} + \Delta C_T.(t - t_{norm}) + \left[\frac{\Delta C_Q^{norm} - \Delta C_Q^{crash}}{t_{norm} - t_{crash}}.(t - t_{norm}) + \Delta C_Q^{norm}\right].(q - q_{norm} - \Delta Q_T.(t - t_{norm}))$$

The proofs of the characteristics mentioned as axioms for the proposed function *TC (t,q)* are presented in the appendix.

To illustrate the proposed model, the cost function of a sample activity is calculated, and depicted in Figure 7. The related parameter of the activity and their descriptions are given in Table 1.

Table 1: Characteristics of a Sample Activity

	Parameter	Description	Value
1	t_{norm}	Normal duration of activity	500
2	t_{crash}	Crash duration of activity	80
3	C_{norm}	Cost of doing activity in normal duration of activity	30
4	C_{crash}	Cost of doing activity in crash duration of activity	-20
5	q_{norm}	Quality of doing activity in normal duration of activity	80%
6	q_{crash}	Quality of doing activity in crash duration of activity	1%
7	Δ_{CnormQ}	Cost of increasing one percent of quality in normal duration of activity	3
8	ΔC_Q^{crash}	Cost of increasing one percent of quality in crash duration of activity	5

The cost function of the sample activity is as follows:

$$TC(t,q) = 500 - 20(t-80) + \left[\frac{-2}{50}.(t-80) + 3\right].(q - 80 - (t-80)) = 2100 - 26.2t + 6.2q + 0.04t^2 - 0.04tq$$

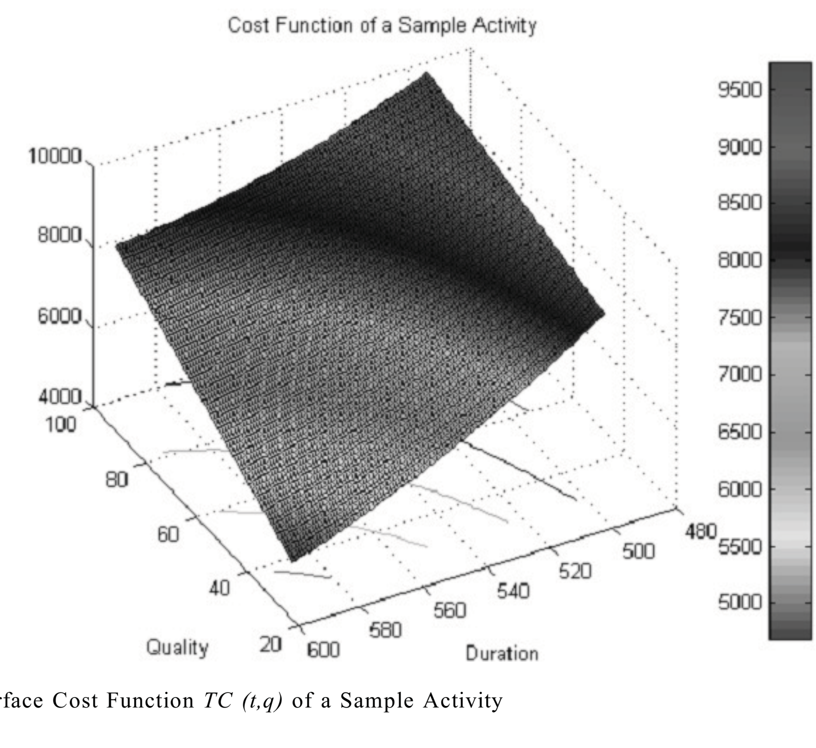

Fig. 7: Surface Cost Function *TC (t,q)* of a Sample Activity

To characterize the cost function of an activity, we need only 8 parameters which are all very easy to obtain. This makes the proposed model very simple to implement for real world applications. In the following section, we develop a new trade-off model for the trio of objectives of a project based on the proposed model.

The Proposed Trade-off Model for the Trio of Objectives of a Project:

Based on the proposed model, we develop a tri-objective mathematical model in this section. A project is defined as a directed acyclic graph *G= (V, E)* in which *V* is the set of nodes and *E* is the set of the arcs. In this case, the project is modeled by an activity-on-node network (AON) where its nodes represent project activities and its arcs are used to define precedents for activities.

Parameters:

n	Number of actual (non-dummy) activities	
A_{n+1}	The dummy sink activity of the project	
$t_{norm,i}$	Normal duration of activity i	$i=1,\ldots,n$
$t_{crash,i}$	Crash duration of activity i	$i=1,\ldots,n$
$C_{norm,i}$	Cost of doing activity in normal duration of activity i	$i=1,\ldots,n$
$C_{crash,i}$	Cost of doing activity in crash duration of activity i	$i=1,\ldots,n$
$q_{norm,i}$	Quality of doing activity in normal duration of activity i	$i=1,\ldots,n$
$q_{crash,i}$	Quality of doing activity in crash duration of activity i	$i=1,\ldots,n$
$\Delta C_Q^{norm,i}$	Cost of increasing one percent of quality in normal duration of activity i	$i=1,\ldots,n$
$\Delta C_Q^{crash,i}$	Cost of increasing one percent of quality in crash duration of activity i	$i=1,\ldots,n$

Decision Variables:

t_i = Duration of activity i $\qquad i=1,\ldots,n$

q_i = Quality of activity i $\qquad i=1,\ldots,n$

c_i = Cost of activity i $\qquad i=1,\ldots,n$

s_i = Start time of activity i $\qquad i=1,\ldots,n$

$$\text{Min } S_{A_{n+1}},\ \text{Min} \sum_{i=1}^{n} c_i,\ \text{Max } \frac{\sum_{i=1}^{n} q_i}{n} \tag{1}$$

$$s_i + t_i \le s_j \qquad \forall (i,j) \in E \tag{2}$$

$$t_{crash,i} \le t_i \le t_{norm,i} \qquad i=1,\ldots,n \tag{3}$$

$$q_{crash,i} \le q_i \le 100\% \qquad i=1,\ldots,n \tag{4}$$

$$\begin{aligned} c_i = C_{norm,i} + \frac{C_{norm,i} - C_{crash,i}}{t_{norm,i} - t_{crash,i}}.(t_i - t_{norm,i}) + \\ \left[\frac{\Delta C_Q^{norm,i} - \Delta C_Q^{crash,i}}{t_{norm,i} - t_{crash,i}}.(t_i - t_{norm,i}) + \Delta C_Q^{norm,i} \right] \times \qquad i=1,\ldots,n \\ (q_i - q_{norm,i} - \frac{q_{norm,i} - q_{crash,i}}{t_{norm,i} - t_{crash,i}}.(t_i - t_{norm,i})) \end{aligned} \tag{5}$$

$$s_i \geq 0 \qquad\qquad i = 1,\ldots,n \tag{6}$$

Model Description:

1 Equation (1) is used to define the objectives. The first objective $S_{A_{n+1}}$, minimizes the makespan (time) of the project (the critical path method determines the makespan of the project). The second one $\sum_{i=1}^{n} c_i$ minimized the total cost of the project, and finally $\frac{\sum_{i=1}^{n} q_i}{n}$ maximizes the total quality of the project, which is defined as the arithmetic mean of qualities of activities.

2 The set of constraints (2) is used for the precedence relationships.

3 The set of constraints (3) is used to constrain the values of duration of activities within the interval $[t_{crash}, t_{norm}]$

4 The set of constraints (4) is used to constrain the values of quality of activities within the interval $[q_{crash}, q_{norm}]$

5 The set of constraints (5) is used to calculate the cost of each activity based on the values of duration and quality selected for that activity. Instead of defining this set of constraint, the second objective of the model can be rewritten as:

$$\sum_{i=1}^{n} c_i = \sum_{i=1}^{n} \left\{ C_{norm\,j} + \frac{C_{norm\,j} - C_{crash\,j}}{t_{norm\,j} - t_{crash\,j}}.(t_i - t_{norm\,j}) + \right.$$
$$\left. \left[\frac{\Delta C_Q^{norm\,j} - \Delta C_Q^{crash\,j}}{t_{norm\,j} - t_{crash\,j}}.(t_i - t_{norm\,j}) + \Delta C_Q^{norm\,j} \right] \times \left(q_i - q_{norm\,j} - \frac{q_{norm\,j} - q_{crash\,j}}{t_{norm\,j} - t_{crash\,j}}.(t_i - t_{norm\,j})\right) \right\}$$

But to increase the legibility of the model, this set is incorporated in the model.

6 Constraints (6) are used as sign constraints.

To illustrate the proposed model, a sample problem is investigated in the coming section.

Numerical Example:

In this section, a sample problem is considered and by which the applicability of the model is demonstrated.

A Sample Problem:

A sample project with 12 real activities (2-13) and two dummy activities (activities 1, and 14) is considered here. The network of the project is illustrated in Figure 8.

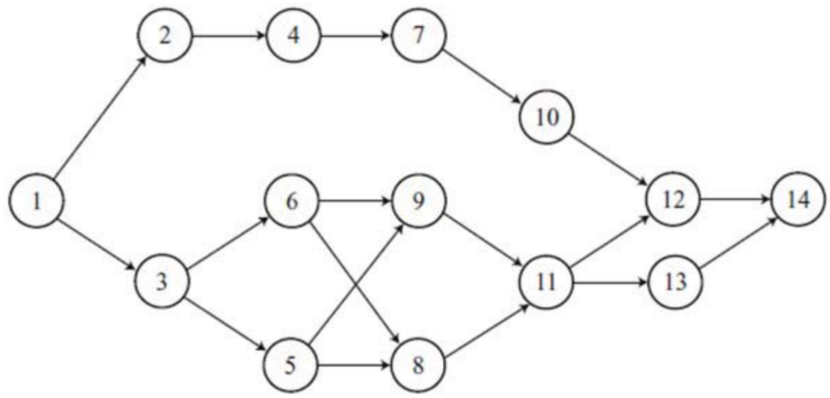

Fig. 8: The Network of the Sample Problem

Supplementary data of the problem is given in Table 2.

Table 2: Data of the Sample Problem

		Activities											
No	Parameter	2	3	4	5	6	7	8	9	10	11	12	13
1	$t_{norm,i}$	213	232	76	233	177	70	106	160	242	243	82	245
2	$t_{crash,i}$	185	114	57	50	78	59	78	139	152	32	64	208
3	$C_{norm,i}$	365	408	97	392	293	82	119	240	475	326	130	300
4	$C_{crash,i}$	359	390	96	349	273	81	117	238	461	300	128	296
5	$q_{norm,i}$	82	79	85	76	80	79	83	83	77	80	80	82
6	$q_{crash,i}$	46	48	48	37	45	31	47	26	34	27	28	50
7	$\Delta C_Q^{norm,i}$	1.6	1.01	0.46	1.11	0.68	0.21	0.44	0.78	1.35	1.45	0.47	1.05
8	$\Delta C_Q^{crash,i}$	7.06	3.83	1.97	4.72	2.64	0.84	1.58	2.77	5.44	6.19	2.09	3.81

Solution Method:

Unlike single-objective problems that there (may) exist a single solution to the optimization problem, a set of efficient solutions can be given for a multi-objective problem. These efficient solutions called Pareto optimal solutions form the Pareto front of a problem. A solution is Pareto optimal or efficient if there exists no feasible solution for which an improvement in one objective does not lead to a simultaneous degradation in one or more of the remaining objectives. Since the problem is multi-objective, and ε-constraint method has several advantages over other related methods such as weighted sum method, we have used it as the solution method. The augmented version of this method presented by Mavrotas (2006) is used here because it produces only efficient solutions, or in other words it avoids the generation of weakly efficient solutions (for more information about the augmented ε-constraint method refer to Mavrotas (2006).

To construct the Pareto optimal front in the ε-constraint method, we optimize one of the objective functions, bounding the other objective functions as constraints. In order to properly apply the ε-constraint method, the range of each of the remaining objective functions that will be used as constraints should be determined. One of the most common ways to determine this range is using the ideal and nadir values of the objectives (the worst possible values of the objective, respectively) as the upper and lower bounds of the range. Then we divide this range into equal intervals using some intermediate equidistant grid points. These grid points are used to vary parametrically the as the right hand side of the according objective function. So if we define n_1 grid points for the first remaining objective and n_2 grid points for the second, a total of n_1,n_2 problems should be solved. To determine the ranges of the objectives, the payoff table, a table with the results from the individual optimization of the three objective functions is used. The Pay-off table of the sample problem is given in Table 3:

It is worth noting that since quality and time of activities are considered as independent variables; optimizing one objective does not affect the values of the other one. Using the augmented ε-constraint method as the solution method, the Pareto front of the sample problem is obtained as illustrated in Figure 9.

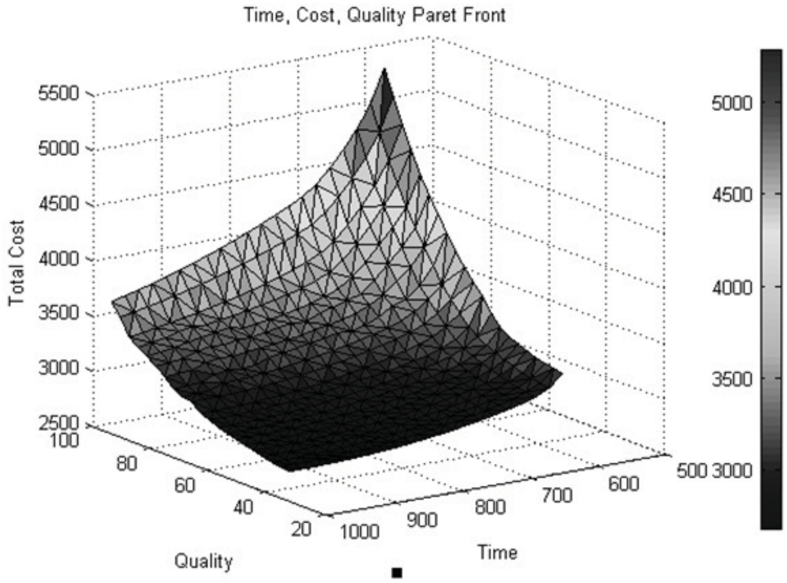

Fig. 9: Pareto Front of the Sample Problem

Table 3: The Pay-off Table of the Sample Problem

The objective to optimize	Values of Objectives		
	Cost	Time	Quality
Cost	2679.549	972.783	38.917
Time	5278.836	571	100
Quality	3120.565	571	38.917
Ideal	2679.549	571	100
Nadir	5278.836	972.783	38.917

Since the Pareto front is three dimensional, to provide insightful information that can help the managers in making trade-off decisions, we can provide them with quality, or cost, or time contours of the surface as illustrated in Figure 10, Figure 11, Figure 12 respectively.

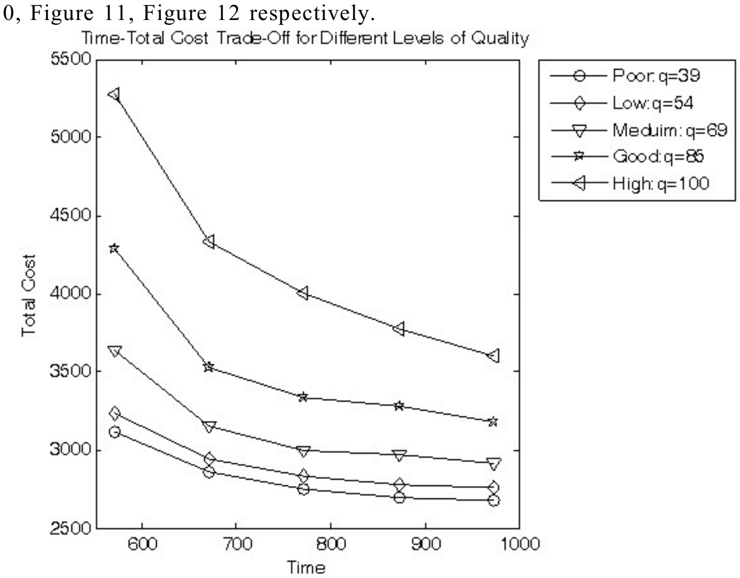

Fig. 10: Time-Total Cost Trade-Off for Different Levels of Quality

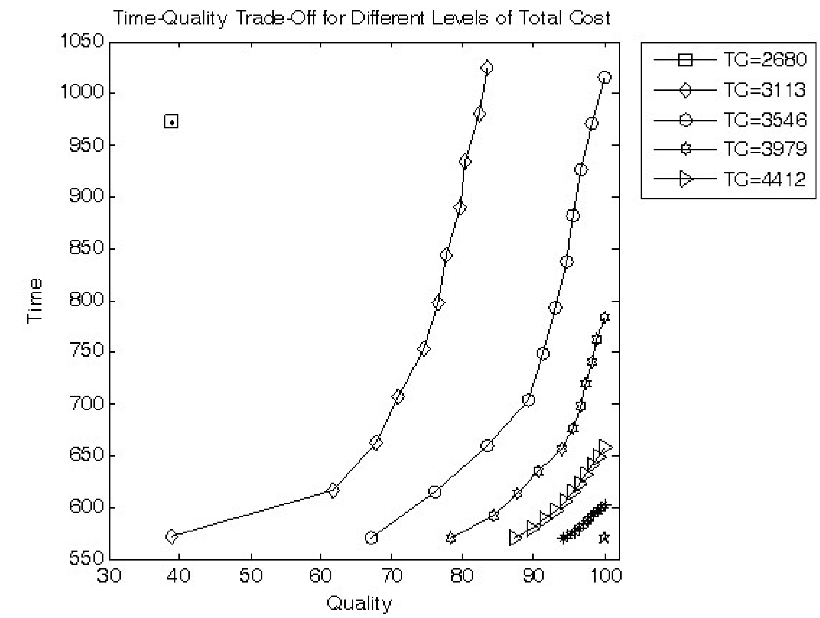

Fig. 11: Time-Quality Trade-Off for Different Levels of Total Cost

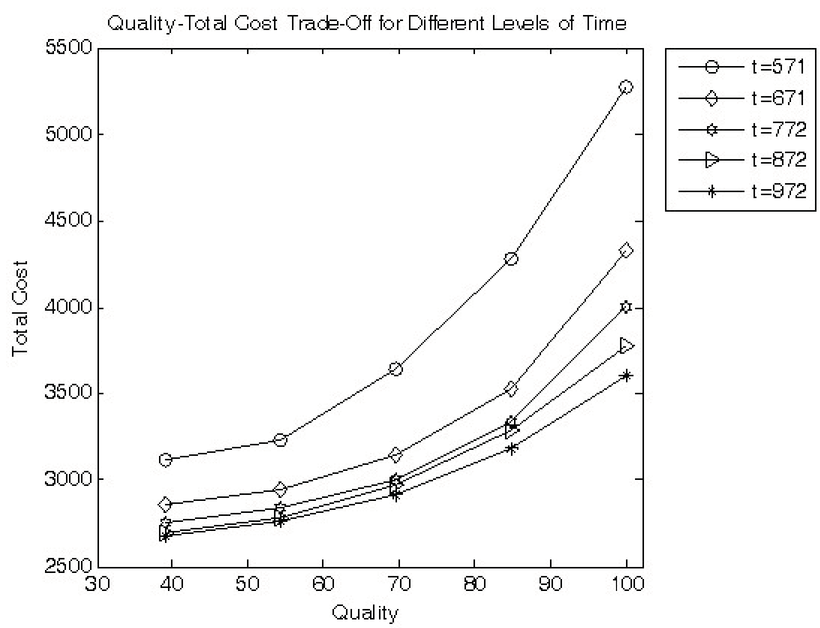

Fig. 12: Quality-Total Cost Trade-Off for Different Levels of Time

Conclusions and Further Suggestions:

In this paper, the existing models to define a true relation between the three features of an activity i.e. time, cost, and quality, were investigated thoroughly. The models found in the literature, lack the generality that an activity can be executed with the least possible time and the best quality. To address this issue, we first proposed general characteristics of a true relation as six axioms. Then we presented a new model that meets the mentioned characteristics, and also is practicable for real world applications. To illustrate the proposed model, the relation of features of a sample activity was demonstrated using this model. Then we developed a new mathematical trade-off model for the trio of objectives of a project based on the proposed model. To show the applicability of the new trade-off model, a sample project with twelve activities was investigated. The according multi-objective decision making model, was solved by a recent version of the ε-constraint method, the augmented ε-constraint method. Using this method, the efficient solution set or Pareto optimal front of the problem was provided. Since the Pareto front of the problem was three dimensional, to provide insightful information that could help the managers in making trade-off decisions, quality, cost, and time contours of the surface were also provided. To investigate the applicability of the proposed model and find the unforeseen pitfalls of the model, studying a real project based on the proposed model is suggested. Since the model can be easily adopted for real world projects, it is also suggested that the model get embedded in a Decision Support System (DSS) to help project managers to make appropriate project scheduling decisions considering quality, time, and cost criteria alongside.

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Appendix

The proofs of the characteristics for the proposed function *TC (t,q)*:

1. C is a non-increasing function of time:

$$\forall t,q:\frac{\partial TC(t,q)}{\partial t}=0+\Delta C_T+\frac{\Delta C_Q^{norm}-\Delta C_Q^{crash}}{t_{norm}-t_{crash}}.(q-q_{norm}-\Delta Q_T.(t-t_{norm}))+$$

$$-\Delta Q_T.\left[\frac{\Delta C_Q^{norm}-\Delta C_Q^{crash}}{t_{norm}-t_{crash}}.(t-t_{norm})+\Delta C_Q^{norm}\right]$$

$$=\Delta C_T+\frac{\Delta C_Q^{norm}-\Delta C_Q^{crash}}{t_{norm}-t_{crash}}.(q-q_{norm})-\Delta Q_T.\Delta C_Q^{norm}-2.\frac{\Delta C_Q^{norm}-\Delta C_Q^{crash}}{t_{norm}-t_{crash}}.\Delta Q_T.(t-t_{norm})$$

$$=\underbrace{\Delta C_T}_{\leq 0}+\underbrace{\underbrace{\frac{\Delta C_Q^{norm}-\Delta C_Q^{crash}}{t_{norm}-t_{crash}}}_{\leq 0}.\underbrace{\left[q-q_{norm}-2\Delta Q_T.(t-t_{norm})\right]}_{\geq 0}}_{\leq 0}+\underbrace{-\underbrace{\Delta Q_T.\Delta C_Q^{norm}}_{\geq 0}}_{\leq 0}\leq 0$$

The relation $q-q_{norm}-2\Delta Q_T.(t-t_{norm})\geq 0$ is proved as follows:

$$q-q_{norm}-2\underbrace{\Delta Q_T}_{\geq 0}.(t-t_{norm})\geq q_{crash}-q_{norm}-2.\underbrace{\Delta Q_T}_{\frac{q_{norm}-q_{crash}}{t_{norm}-t_{crash}}}.(t_{crash}-t_{norm})=q_{norm}-q_{crash}\geq 0$$

2. C is a non-decreasing function of quality:

$$\forall t,q:\frac{\partial TC(t,q)}{\partial q}=0+0+\left[\underbrace{\frac{\Delta C_Q^{norm}-\Delta C_Q^{crash}}{t_{norm}-t_{crash}}}_{\leq 0}.(t-t_{norm})+\Delta C_Q^{norm}\right]\geq$$

$$\left[\frac{\Delta C_Q^{norm}-\Delta C_Q^{crash}}{t_{norm}-t_{crash}}.(t_{norm}-t_{norm})+\Delta C_Q^{norm}\right]=\Delta C_Q^{norm}\geq 0$$

3. C is a convex function of t in the vicinity of *TC (t,q)*

$$\forall t,q:\frac{\partial^2 TC}{\partial t^2}=\frac{\partial}{\partial t}\left(\frac{\partial TC}{\partial t}\right)=$$

$$\frac{\partial}{\partial t}\left(\Delta C_T+\frac{\Delta C_Q^{norm}-\Delta C_Q^{crash}}{t_{norm}-t_{crash}}.\left[q-q_{norm}-2\Delta Q_T.(t-t_{norm})\right]-\Delta Q_T.\Delta C_Q^{norm}\right)=$$

$$=0+-2\underbrace{\Delta Q_T}_{\geq 0}\underbrace{\frac{\Delta C_Q^{norm}-\Delta C_Q^{crash}}{t_{norm}-t_{crash}}}_{\leq 0}\geq 0$$

4. C is convex function of q in the vicinity of $q=q_{\min},100\%$

$$\forall t,q:\frac{\partial^2 TC}{\partial q^2}=\frac{\partial}{\partial q}\left(\frac{\partial TC}{\partial q}\right)=\frac{\partial}{\partial q}\left(\frac{\Delta C_Q^{norm}-\Delta C_Q^{crash}}{t_{norm}-t_{crash}}.(t-t_{norm})+\Delta C_Q^{norm}\right)$$

$$=\frac{\Delta C_Q^{norm}-\Delta C_Q^{crash}}{t_{norm}-t_{crash}}\leq 0$$

5. $\frac{\partial^2 C}{\partial q\,\partial t}\leq 0$:

$$\forall t,q:\frac{\partial^2 C}{\partial q\,\partial t}=\frac{\partial^2 C}{\partial t\,\partial q}=\frac{\partial}{\partial t}\left(\frac{\partial TC}{\partial q}\right)=\frac{\partial}{\partial t}\left(\frac{\Delta C_Q^{norm}-\Delta C_Q^{crash}}{t_{norm}-t_{crash}}.(t-t_{norm})+\Delta C_Q^{norm}\right)$$

$$=\frac{\Delta C_Q^{norm}-\Delta C_Q^{crash}}{t_{norm}-t_{crash}}\leq 0$$