Australian Journal of Basic and Applied Sciences, 2(3): 500-509, 2008
ISSN 1991-8178

Axisymmetric Gravitational Oscillation of a Fluid Cylinder Under Longitudinal Oscillating Electric Field

[1]Ahmed E. Radwan, [2]Samia S. Elazab and [2]Zeinab M. Ismail
[1]Department of Mathematics, Faculty of Science, Ain- Shams University, Abbassia, Cairo, Egypt.
[2]Mathematics Department, Women's University College, Ain-Shams University,
Asmaa Fahmy Street, Heliopolis, Cairo, Egypt.

Abstract: The gravitational oscillation of a dielectric fluid cylinder surrounded by gravitational dielectric medium of negligible motion has been investigated via the normal mode analysis for axisymmetric perturbation. The acting forces on the model are : self-gravitating, pressure gradient and electrodynamic forces with oscillating time dependent electric field. The model is governed by Mathieu second order integro-differential equation. The oscillating electric field is only destabilizing in few axisymmentric states but it is strongly stabilizing in the remaining axisymmentric states. The self gravitating has strong destabilizing influence in the domain 0<x<1.0667 while it is stabilizing in the other states. The oscillating electric fields modify the gravitation stability states.

Key words: Electrogravitational, Periodic time dependent, Hydrodynamic stability, Stability of laminar flows, PACS numbers: 47.20.-K Hydrodynamic stability, 47.15.Fe Stability of laminar flows

INTRODUCTION

Rayleigh (1945) has been the response of the capillary instability and oscillation of a long fluid cylinder. The stability criterion founded by Rayleigh (1945). He laid the theoretical foundation for treating this problems. More extension along this problems and other acting by different forces are studied by Chandrasekhar (1981). Recently, Radwan (2004 and 2005)has developed the hydrodynamics and hydromagnetic instability of different cylindrical models. The electro hydrodynamics stability of cylindrical interface has been investigated in several contexts (Reynolds 1965, Yih 1968, Nayyar and Nurty 1960, Mohammed et.al. 1986 and Baker 1983). These works have some applications concerning from design of sprays to the design of inkjet printers. Chandrasekhar and Fermi (1953) has, for first time, studied the self-gravitating instability of fluid cylinder. See also Chandrasekhar (1981). The electrogravitational stability of fluid cylinder has developed by Radwan (1991). He considered that the fluids are penetrated by constant and uniform electric fields. In the present paper we study the axisymmetric gravitational oscillation of a fluid cylinder under longitudinal oscillating time-dependent electric field. We find that the unperturbed state involves parameters which are time-depentent while the elimination of the time from the fundamental equations is cumbersome. We obtained second order differential equation of Mathieu, cf. Mclachlan (1964), Morse and Feshbach (1953), Woodson & Melcher (1968). The details and the characteristics of the in-stability domains has been obtained with using the normal mode analysis.

Basic Equations:

We consider an incompressible, gravitational, inviscid fluid cylinder, of (radius R_0) uniform mass density ρ and dielectric constant ε^i surrounded by a dielectric medium of negligible motion with dielectric constant ε^c (superscripts i and e denote respectively the interior and exterior of the fluid cylinder). We assume that the quase-static approximation, (see Baker 1983, Mohamed 1986 & Radwan 1991 a,b), is valid and also that there are initially no surface charges at the interfaces so that the surface charge density will be postulated to be zero during the perturbation. The fluid cylinder and the surrounding region are pervaded by the oscillating time-dependent electric fields

$$\underline{E}_0^{i,e} = (0,0,E_0 \cos \omega t) \qquad (1)$$

Corresponding Author: Ahmed E. Radwan, Department of Mathematics, Faculty of Science, Ain- Shams University, Abbassia, Cairo, Egypt.

where E_0 is the amplitude of the electric fields inside and outside the fluid jet, t is the time and ω is the electric field frequency. The components of $\underline{E}_0^{i,e}$ are taken along the utilized cylindrical polor coordinates *(r,0,z)* with the z-axis coinciding with the axis of the cylinder. The forces acted on the fluid cylinder are the pressure gradient, self- gravitating and electrodynamic forces while the medium surrounding the fluid cylinder is subject up on electrodynamic and self- gravitating forces only.

The required basic equations for studying the stability of such kind of problems are coming out from the combination of the ordinary hydrodynamic equations together with those of Maxwell's electrodynamic theory and with those of Newtonian's gravitational field. Under the present circumstances the basic equations are the vector electrogravitational equation of motion (2), the fluid conservation of mass equation (3), the solenoidal character of the electric displacement current equation (4), the circulation of the electric field (note that there is no surface charge) equation (5), and the poisson's and laplace's equations satisfying the gravitational potentials interior and exterior the fluid cylinder, equation (6) and (7).
These equations may be formulated as

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\underline{u}.\underline{\nabla}) \tag{3}$$

$$\underline{\nabla}.\underline{u}^i = 0 \tag{4}$$

$$\underline{\nabla}.(\varepsilon\,\underline{E})^{i,e} = 0 \tag{5}$$

$$\underline{\nabla} \wedge \underline{E}^{i,e} = 0 \tag{6}$$

$$\nabla^2 V^i = -4\pi\, \rho^i\, G \tag{7}$$

$$\nabla^2 V^e = 0 \tag{8}$$

where $\underline{u}$ and p are the fluid velocity vector and kinetic pressure, $\underline{E}$ is the electric field intensity, V is the gravitational potential and G is the gravitational constant.

Unperturbed State:

In the initial state, the basic equations take the form

$$\underline{\nabla}\Pi_0 = 0\ , \qquad \Pi_0 = p_0^i - \rho V_0^i - \frac{1}{2}\varepsilon^i(\underline{E}.\underline{E})^i = const \tag{9,10}$$

$$\underline{\nabla}.\underline{u}_0^i = 0\ , \qquad \underline{\nabla}.(\varepsilon\,\underline{E}_0)^{i,e} = 0 \tag{11,12}$$

$$\underline{\nabla} \wedge E^{i,e} = 0, \qquad \nabla^2 V_0^i = -4\pi G \rho^i \tag{13,14}$$

$$\nabla^2 V_0^e = 0 \tag{15}$$

where the subscript 0 here and henceforth indicates unperturbed quantities. The equations (9)----(15) are simplified, by taking into account, $\frac{\partial}{\partial z} = 0$, $\frac{\partial}{\partial \varphi} = 0$ are solved and moreover the solutions are matched at the unperturbed boundary surface $r=R_0$.

Apart from the singular solutions as r tends to zero inside the fluid cylinder and as r tends to infinity exterior it, the solution of the basic equations (1)----(8) in the initial state are given by

$$V_0^i = -\pi G \rho\, r^2 \tag{16}$$

$$V_0^e = 2\pi G \rho R_0^2 \log(\frac{R_0}{r}) - \pi G \rho R_0^2 \tag{17}$$

$$p_0^i = \pi \rho^2 G(R_0^2 - r^2) + c \tag{18}$$

with

$$p_0^i = c, \qquad\qquad at \;\; r = R_0 \tag{19}$$

Perturbation State:

Consider the influence of a small wave disturbance on the boundary surface of the fluid propagation in the positive z-direction. The surface deflection at time t is assumed to be of the form

$$r = R_0 + R_1 \tag{20}$$

with

$$R_1 = \gamma(t)\exp\left(i(kz)\right) \tag{21}$$

where *γ(t)* is the amplitude of the perturbation which is some function of t, while k (a real number)is the longitudinal wavenumber. The second term on the right side of (20), R_1 is the surface-wave elevation measured from the unperturbed position. For small departures from the initial unperturbed state, each physical quantity *q(r, 0, z, t)* may be expanded as

$$q(r,0,z,t) = q_0(r) + \gamma(t)\, q_1(r,0,z) \tag{22}$$

where q is pertaining to $p, \underline{u}, V^{i,e}, \underline{E}^{i,e}$ and subscript 1 associating the perturbed quantities. Based on this expansion, the basic equations (2)----(8) yield the linearised equations in the fluid

$$\frac{\partial \underline{u}_1^i}{\partial t} = -\underline{\nabla}\Pi_1^i, \qquad\qquad \Pi_1^i = \frac{p_1^i}{\rho^i} - V_1^i - \left(\frac{\varepsilon^i}{2\rho}\right)(\underline{E}^i . \underline{E}^i) \tag{23,24}$$

$$\underline{\nabla}.(\varepsilon \underline{E}_1)^i = 0 \;, \qquad\qquad \underline{\nabla} \wedge E_1^i = 0 \tag{25,26}$$

$$\underline{\nabla}\underline{u}_1^i = 0, \qquad\qquad \nabla^2 V_1^i = 0 \tag{27,28}$$

Surrounding the Fluid Cylinder:

$$\underline{\nabla}.(\varepsilon^e E_1^e) = 0 \;, \qquad\qquad \underline{\nabla} \wedge E_1^e = 0 \tag{29,30}$$

$$\nabla^2 V_1^e = 0 \tag{31}$$

One have to refer here that $\rho(\Pi_1^i + V_1^i)$ (in view of equation (24)) represents the tolal electrohydrodynamic pressure, which is the sum of the kinetic and electrodynamic pressures.

By taking the divergence of the vector equation (23) and utilizing the conservation mass equation (27) we get

$$\nabla^2 \Pi_1^i = 0 \tag{32}$$

Equation (26) as well as (30) means that $\underline{E}_1^{i,e}$ can be derived from a scalar (electrical potential) function $\Psi_1^{i,e}$ such that

$$\underline{E}_1^{i,e} = -\underline{\nabla}\Psi_1^{i,e} \tag{33}$$

Combining equation(33) with equations (25) and (29), we get

$$\nabla^2 \Psi_1^{i,e} = 0 \tag{34}$$

As we see the perturbed linearised variables could be obtained if Laplace's equations (28), (31), and (34) are solved for the scalar functions $V_1^{i,e}$ *and* $\Psi_1^{i,e}$

By the use of the linear perturbation technique for cylindrically time space dependence, each relevant perturbation quantity $q_1(r,0,z,t)$ may be expressed as

$$q_1(r,0,z,t) = \gamma(t)\, q_1^*(r) \exp(i(kz)) \tag{35}$$

Consequently Laplace's equations (28), (31), (32) and (34)

$$(r^{-1}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r}) + r^{-1}\frac{\partial}{\partial z}(r\frac{\partial}{\partial z}))q_1(r,0,z,t) = 0 \tag{36}$$

could be simplified and turned to ordinary total second order differential equation

$$r^{-1}\frac{d}{dr}(r\frac{d}{dr}q_1(r)) - k^2 q_1(r) = 0$$

Apart from the singular solutions as $r \to 0$ inside the fluid cylinder and as $r \to \infty$ outside the cylinder in the surrounding medium, the non-singular solutions are obtained

$$V_1^i = A^i(t)\, \gamma(t)\, I_0(kr) \exp(i(kz)) \tag{38}$$

$$V_1^e = A^e(t)\, \gamma(t)\, K_0(kr) \exp(i(kz)) \tag{39}$$

$$\Psi_1^i = B^i(t)\; \gamma(t)\, I_0(kr) \exp(i(kz)) \tag{40}$$

$$\Psi_1^e = B^e(t)\; \gamma(t)\, K_0(kr) \exp(i(kz)) \tag{41}$$

$$\Pi_1^i = c^i(t)\; \gamma(t)\, I_0(kr) \exp(i(kz)) \tag{42}$$

where A^i, A^e, B^i, B^e *and* c are arbitrary functions of integrations to be determined, $I_0(kr)$ *and* $K_0(kr)$ are the modified Bessel's functions of the first and second kind of order m.

Boundary Conditions:

The non-singular solutions of the linearised perturbation equations (23)---(31) of the basic equations (2)----(8) given by the system (38)----(42) must satisfy certain appropriate boundary conditions.

(i) The normal component of the velocity vector must be compatible with the velocity of the boundary perturbed surface of the fluid at the initial level $r = R_0$. This condition yields

$$u_{1r} = \frac{\partial r}{\partial t} \qquad at \qquad r = R_0 \tag{43}$$

By the use of equations (20), (23), (24) and (42) for the condition (43), we get

$$c^i(t) = \frac{-\rho R_0}{x\gamma(t) I_0'(x)} \frac{d^2\gamma}{dt^2} \tag{44}$$

where $x\,(=kR_0)$ is the dimensionless longitudinal wavenumber

(ii) The gravitational potential $V\,(=V_0+\gamma(t)V_1)$ and its derivative must be continuous across the perturb boundary fluid interface at $r=R_0$. By the use of these conditions which are

$$V_1^i - V_1^e = R_1\left(\frac{\partial V_0^e}{\partial r} - \frac{\partial V_0^i}{\partial r}\right) \tag{45}$$

$$\frac{\partial V_1^i}{\partial r} - \frac{\partial V_1^e}{\partial r} = R_1\left(\frac{\partial^2 V_1^e}{\partial r^2} - \frac{\partial^2 V_1^i}{\partial r^2}\right) \tag{46}$$

and upon utilizing equations (16), (17), (21), (38) and (39), we obtain

$$V_1^i = 4\pi G\rho R_0 K_0(x)\gamma(t) I_0(kr)\exp(i(kz)) \tag{47}$$

$$V_1^e = 4\pi G\rho R_0 I_0(x)\gamma(t) K_0(kr)\exp(i(kz)) \tag{48}$$

where use has been made of the Wronskian relation

$$W_0(I_0(x), K_0(x)) = I_0(x)K_0'(x) - I_0'(x)K_0(x) = -x^{-1} \tag{49}$$

in obtaining (47) and (48).

(iii) The normal component of the electric displacement current must be continuous across the perturbed boundary interface at $r=R_0$. This condition read

$$\underline{N}.(\varepsilon^i \underline{E}^i - \varepsilon^e \underline{E}^e) = 0 \tag{50}$$

$$\underline{E} = \underline{E}_0 + R_1\frac{\partial \underline{E}_0}{\partial r} + \underline{E}_1 \tag{51}$$

Here $\underline{N}$ is, the outward unit vector normal to the interface (20) at $r=R_0$, given by

$$\underline{n} = \frac{\nabla f(r,z,t)}{|\nabla f(r,z,t)|} \tag{52}$$

$$f(r,z,t) = r - R_0 - R_1 \tag{53}$$

$$\underline{n}_0 = (1,0,0)$$
$$\underline{n}_1 = (0,0,-ik)\gamma(t)\exp i(kz) \tag{54}$$

Consequently the condition (50), yields

$$\varepsilon^i I_0'(x)B^i - \varepsilon^e K_0'(x)B^e = iE_0(\varepsilon^e - \varepsilon^i)\cos\omega t \tag{55}$$

(iv)The electric potential Ψ must be continuous across the perturbed boundary surface of the fluid cylinder at the initial level $r = R_0$, i.e.

$$B^i(t)\, I_0(x) = B^e(t)\, K_0(x) \tag{56}$$

Equations (55) and (56), finally yield

$$B^i(t) = \left(\frac{K_0(x)}{I_0(x)}\right) B^e(t) \tag{57}$$

with

$$B^e(t) = \frac{iE_0(\varepsilon^e - \varepsilon^i) I_0(x)}{(\varepsilon^i I_0'(x) K_0(x) - \varepsilon^e I_0(x) K_0'(x))} \cos \omega t \tag{58}$$

(v) The stresses across the cylindrical fluid interface are being due to the fluid kinetic pressure, self-gravitating and electrical forces. All these are well known since a time ago except the latter which become familiar after the pioneering works of Reynolds (1965). See also Yih (1968).
The stresses due to the electrical forces are, in the tensor form, being

$$\sigma_{fj} = E_f E_j - \frac{1}{2} E^2 \left(\varepsilon - \rho \left(\frac{\partial \varepsilon}{\partial \rho}\right)_T\right) \delta_{fj}$$

Here T is the temperature, ε is the fluid permittivity and E the electric field strength. Following Reylnolds (1965), we denote the quantity $\rho\left(\frac{\partial \varepsilon}{\partial \rho}\right)_T$ by τ, the following stresses continuity relation exists at the interface

$$\left\langle p^i - \frac{1}{2}(\tau^i + \varepsilon^i)(\underline{E}^i . \underline{E}^i) - \rho^i V^i \right\rangle = 0$$

This jump restriction yields

$$\Pi^i + \frac{1}{2} \varepsilon^i (\underline{E}^i . \underline{E}^i) + \rho^i V^i = \frac{1}{2} \varepsilon^e (\underline{E}^e . \underline{E}^e) \tag{59}$$

which is valid across the displaced interface $r = R_0 + \gamma(t) \exp(i(kz))$ By subst
$p_1^i,\ p_0^i,\ \underline{E}_0^{i,e},\ \underline{E}_1^{i,e},\ and\ \ R_1$ in equation (59), we get

$$\frac{d^2\gamma}{dt^2} + \Big[\Big(4\pi G\rho \frac{x I_0'(x)}{I_0(x)}\Big(\frac{1}{2} - I_0(x) K_0(x)\Big) + \frac{E_0^2}{\rho R_0^2} \frac{x^2 I_0'(x) K_0(x) (\varepsilon^i - \varepsilon^e)^2}{(\varepsilon^i I_0'(x) K_0(x) - \varepsilon^e I_0(x) K_0'(x))} \cos^2 \omega t\Big]\gamma(t) = 0 \tag{60}$$

Equation (60) is an integro-differential equation governing the surface displacement $\gamma(t)$. Through which we could identify the instability and stability state, and also identify the self-gravitating and electrodynamic forces influences on the stability of the present model.

However in order to do so, it is found more convenient to express this equation in the following form

$$[\frac{d^2}{d\eta^2}+(b-h^2\cos^2\eta)]\gamma(t)=0\ , \qquad \eta=\omega t \tag{61}$$

where

$$b=\frac{4\pi G\rho}{\omega^2}\frac{xI_0'(x)}{I_0(x)}(\frac{1}{2}-I_0(x)K_0(x)) \tag{62}$$

$$h^2=-\frac{E_0^2}{\omega^2 R_0^2\rho}\frac{x^2(\varepsilon^i-\varepsilon^e)^2 I_0'(x)K_0(x)}{[\varepsilon^i I_0'(x)K_0(x)-\varepsilon^e I_0(x)K_0'(x)]} \tag{63}$$

Equation (61) has the canonical form

$$[\frac{d^2}{d\eta^2}+(a-2q^*\cos 2\eta)]\gamma(t)=0 \tag{64}$$

with

$$q^*=\frac{h^2}{4}, \qquad a=b-\frac{h^2}{2} \tag{65}$$

Equation (64) is the Mathieu differential equation. The properties of the Mathieu functions are explained and investigated by Mclachlan (1964). The solutions of equation (64), under appropriate restrictions, could be periodic and consequently the considered model will be stable and vice verse. The conditions required for periodicity of Mathieu functions is mainly depond on the correlation relates the parameters *a and q**. However it is well known, see Mclachlan (1964), that (a , q^*) -plane is divided essentially into two stable and unstable domains separated by the characteristic curves of Mathieu function. Thence we can state as a general statement that a solution of Mathieu integro-differential equation is unstable if the point (a , q^*) say, in the (a , q^*) -plane lies interior an unstable domain, otherwise it is stable.

Discussions:

The appropriate solutions of equation (64) are given in terms of what called ordinary Mathieu functions which, indeed, are periodic in time t with period π or 2π

Corresponding to extremely small values of q^*, the first region of instability is bounded by the curves

$$a=\pm q^*+1 \tag{66}$$

The conditions for oscillation lead to the problem of the boundary regions of Mathieu functions where Mclachian (1964) gives the condition of stability as

$$\left|\Delta(0)\sin^2(\frac{\pi a}{2})\right|^{\frac{1}{2}}\leq 1 \tag{67}$$

where Δ (0) is the Hill's determinant.

An approximate criterion for the stability near the neighbourhood of the first stable domains of the Mathieu stability domains given by Morse and Feshbach (1953) which is valid only for small values of h^2 *or* q^* i.e. The frequency ω of the electric field is very large.

This criterion, under the present circumstances, states that the model is ordinary stable if the restriction

$$h^4 - 16(1-b)h^2 + 32b(1-b) \geq 0 \tag{68}$$

is satisfied where the equality is corresponding to the marginal (neutral)stability. The inequality (68) is a quadratic relation in h^2 and could be rewritten in the form

$$(h^2 - \alpha_1)(h^2 - \alpha_2) \geq 0 \tag{69}$$

where α_1 *and* α_2 are, the two roots of the equality of the relation (68),being

$$\alpha_1 = 8(1-b) - \Delta \tag{70}$$

$$\alpha_2 = 8(1-b) + \Delta \tag{71}$$

$$\Delta^2 = 32(1-b)(2-3b) \tag{72}$$

The magnetogravitational stability and instability investigation analysis should be carried out in the following different cases

$$(i)\ 0 < b < \frac{2}{3} \qquad (ii)\ \frac{2}{3} < b < 1$$

The case: $0 < b < \frac{2}{3}$

In this case Δ^2 is positive and therefore the two roots α_1 *and* α_2 of the equality (68) are real. Now we will show that both α_1 *and* α_2 are $\alpha_1 \neq +ve$ α_1 then must be negative and this mean

$$\begin{aligned} &8(1-b) \leq b \\ &64(1-b)^2 \leq 32(1-b)(2-3b) \end{aligned} \tag{73}$$

$$\begin{aligned} &\textit{from which we get} \\ &2b \geq 3b \end{aligned} \tag{74}$$

and this is contradiction, so α_1 must be positive and consequen $\alpha_2 \geq 0$ as well ($\alpha_2 > \alpha_1$)t This means that both the quantities $(h^2 - \alpha_1)$ *and* $(h^2 - \alpha_2)$ are negative and that in turn sho inequality (68) is identically satisfied in the axisymmetric disturbance mode $m = 0$.

The Case: $\frac{2}{3} < b < 1$

In this case in which $b < 1$ and simultaneously $3b < 2$, it is found that Δ^2 is negative i.e. Δ is imaginary, therefore the two roots α_1 *and* α_2 are complex. We may prove that the inequality (68) is satisfied as follows. Let $h^2 = -c$ *and* $\alpha_{1,2} = c_1 - ic_2$ *where* c, c_1 *and* c_2 are real, so

$$\begin{aligned} (h^2 - \alpha_1)(h^2 - \alpha_2) &= [-c - (c_1 + ic_2)][-c - (c_1 - ic_2)] \\ &= c^2 + 2cc_1 + c_1^2 + c_2^2 \\ &= (c + c_1)^2 + c_2^2 = +ve \end{aligned} \tag{75}$$

which is positive definite.

By an appeal to the cases (i) and (ii), we deduce that the model is stable under the restrictions

$$0 < b < 1 \tag{76}$$

This means that the model is stable if there exists a critical value ω_0 of the electric field frequency ω such that $\omega > \omega_0$ where ω_0 is given by

$$\omega_0^2 > (4\pi G\rho \frac{xI_0'(x)}{I_0(x)}(\frac{1}{2} - I_0(x)K_0(x)) > 0 \tag{78}$$

one has to mention here that if ω = 0 and at the same time $E_0 = 0$ and we suppose that

$$\gamma(t) = (const)\ \exp(\sigma t) \quad , \tag{79}$$

the second order integro-differential equation of Mathieu equation (60) yields

$$\sigma^2 = 4\pi G\rho \frac{xI_0'(x)}{I_0(x)}[I_0(x)K_0(x) - \frac{1}{2}] \tag{80}$$

where σ is the temporal amplification and note by the way that $(4\pi G\rho)^{-\frac{1}{2}}$ has a unit of time. The relation (80) for the single sausage mode $m = 0$ reduce to the gravitational dispersion relation derived for first time by Chandrasekhar and Fermi (1953). In fact they (1953) have used a totally different technique rather than that used here. They have used the method of representing the solenoidal vectors in term of peloidal and toroidal vector fields, which is valid only for the axisymmentric mode $m = 0$.

To determine the effect of ω it is found more convenient to investigate the eigenvalue relation (80) since the right side of it is the same as the middle side of (78).

Taking into account the recurrence relations (cf. Abramowitz and Stegun 1970) of the modified Bessel's functions and their derivatives
We see, for $x \neq 0$, that

$$x(\frac{I_0'(x)}{I_0(x)}) > 0 \tag{81}$$

$$(I_0(x)\,K_0(x)) > 0 \tag{82}$$

Now, returning to the relation (80), we deduce that the determining of the sign $\frac{\sigma^2}{(4\pi G\rho)}$ is identified if the sign of the quantity

$$Q_0(x) = (I_0(x)K_0(x) - \frac{1}{2}) \tag{83}$$

Here it is found that the quantity $Q_0(x)$ may be positive or negative dependi $x \neq 0$ values. Numerical investigations and analysis of the relation (80) reveal that σ^2 is positive for small values of x while it is negative in all other values of x. In more details: it is unstable in the domain $0 < x < 1.0668$ while it is stable in the domains $1.0668 \leq x < \infty$ where the equality is corresponding to the marginal stability.

From the foregoing discussions, investigations and analysis, we conclude (on using (83) for (80)) that the quantity

$$M^2 = \frac{xI_0'(x)}{I_0(x)}(I_0(x)K_0(x) - \frac{1}{2}) , \qquad M = \frac{\sigma}{(4\pi G\rho)^{\frac{1}{2}}} \tag{84}$$

has the following properties

$$\begin{aligned} &M^2 < 0 \qquad as \qquad in\ the\ ranges \qquad 1.0668 \le x < \infty \\ &M^2 > 0 \qquad as \qquad in\ the\ ranges \qquad 0 < x < 1.0668 \end{aligned} \tag{85}$$

Now returning to the relation (78) concerning the frequency ω_0 of the periodic electric field

$$\frac{\omega_0^2}{(4\pi G\rho)} > (\frac{xI_0'(x)}{I_0(x)}(\frac{1}{2} - I_0(x)K_0(x))) > 0 \tag{86}$$

Therefore, as a general conclusion, we deduce that the electrodynamic force (with a periodic electric field) has stabilizing influence could predominate and overcoming the self-gravitating destabilizing influence of the dielectric fluid cylinder dispersed in a dielectric medium of negligible motion.

However, the self-gravitating destabilizing influence could not be suppressed whatever is the greatest value of the magnitude and frequency of the periodic electric field because the gravitational destabilizing influence will persist.

REFERENCES

Abramowitz, M. and I Stegun, 1970. " Handbook of Mathematical Functions" (Dover Publ., New York, N.Y.), pp: 376.

Baker, L., 1983. Phys. Fluids. 26, 39. See also Yeh, H. 1985. J. Fluid Mech. 152: 479.

Chandrasekhar, S. and E. Fermi, 1953. Astrophys. J., 118: 116.

Chandrasekhar, S., 1954. Astrophys. J., 119: 7.

Chandrasekhar, S., 1981. "Hydrodynamic and Hydromagnetic Stability" Dover Publ., N.Y. See also Robert, P.H. 1967. " An Introduction to MHD." Longman, London.

Gradshteyn, I. and I. Ryzhik, 1980. "Tables of Intergrals, Series and Products ". Academic Press Inc., London.

Morse, P.M. and H. Feshbach, 1953. " Methods of theoretical Phys." Part I. Mc Graw-Hill, N.Y.

Mclachlan, N.W., 1964. " Theory and Applications of Mathieu Functions." Dover Publ., N.Y.

Mohammed, A., *et al.* 1985. J. Math. Phys. 26, 2072. See also 1986. J. Chem. Phys. 85, 445 and 1986a. Quart. Appl. Math. 43: 483.

Nayyar, N.K. and G.S. Murty, 1960. Proc. Phys. Soc. London. 75: 369. See also Larraza, A. and Putterman, S. 1984. J. Fluid Mech., 148: 443.

Plateau, A.J., 1827. " Stattique Experimantle et Theorique des Liquids Soumes aux seule Moleculaires." Vols I and II. Gauther-Villars, Paris, France.

Radwan, A.E, 1991a. J. Plasma Phys., 44: 455.

Radwan, A.E, 1991b. J. Magn. Magn. Matrs., 94: 141.

Radwan, A.E, 2004. Mechanics and Mechanical Engineering, 7: 153.

Radwan, A.E., 2005. J. Applied Maths and Comput., 160: 213.

Rayleigh, J.W., 1945. " The Theory of Sound ." Dover Publ., N.Y.

Woodson, H. and J. Melcher, 1968. " Electromechanical Dynamics." John Willey, Inc.

Yih, C.S., 1968. Phys. Fluids., 11: 1447.